Appendix B Web Appendix

B.1 Extensions and Robustness

B.1.1 Designing a profit maximizing ratings system

So far we studied how different features of a marketplace environment affects the informativeness of ratings and surplus. In practice, rating systems are typically designed by online platforms, and many platforms constantly tweak their ratings system. We now investigate how a platform would actually design its ratings system. To do so, we introduce some modifications to our base model.

Suppose there is a unit mass of consumers who are heterogeneous only in their outside option to joining the platform. This outside option is uniformly distributed between $[0, 1]$. We denote the mass of consumers who join the platform as n_b. Further, we call the per transaction consumer benefit u_b. Consumers choose to join the platform and are then randomly assigned (with equal probability) to the first period or the second period. This captures that consumers use the platform both when there are more or less ratings available to guide their choices.

We suppose that there exists a unit mass of sellers that are monopolists in a product category. Sellers are ex-ante homogeneous and face an outside option of $\bar{v}_s$. We denote the mass of sellers who join the platform by n_s and call π_s the per transaction revenue of the seller. We assume that sellers face some additional platform specific marginal cost of selling on the platform, t.[34] To simplify illustration, we suppose that sellers join the platform prior to period 1 of our main model and therefore do not have information over their true quality when deciding to join the platform.[35]

To maximize profits, the platform makes two choices. First, the platform sets a royalty, r, which it charges sellers. This gives the platform a share r of the sellers' revenue. Second, the platform chooses how easy it is for buyers to leave a rating, i.e. the platform sets e. We assume that the platform can choose any $e \in [0, \bar{e}]$. To simplify exposition, we assume $\bar{e} \leq e^{cs}$. This implies that a larger effort always increases u_b and reduces π_s. The platform makes these decisions prior to period 1 in the base model.

[34] This cost captures the difference in cost of selling on a platform rather than direct to consumers. Reflecting costs in addition to the platforms ad valorem fees. For instance, on Amazon, in addition to the ad valorem fees, there are additional charges for fulfilled by Amazon and other transaction fees.

[35] This assumption captures the importance of relative quality of products on a marketplace, and sellers learning their true relative quality after joining the marketplace. This assumption also allows us to abstract away from seller selection by the platform and focus on the role that the platform plays in influencing ratings.

The effort choice e captures that the platform designs a ratings system to make it easier or more difficult for consumers to leave a rating. For example, to facilitate ratings, a platform can introduce a one-click ratings system or provide users a link to go directly to the ratings page, automated ratings[36], etc. Conversely, a platform can make rating more costly for consumers if it introduces additional authentication and verification steps—such as proof of identity, proof of purchase[37], multiple ratings components—such as the use of multi-dimensional ratings (Schneider et al., 2021), or requiring a written review along with the rating[38].

Proposition 6. *Platforms design a ratings environment that favours sellers ($e^* = e^s = 0$) if and only if $\pi_s - t < u_b$ at $e = 0$.*

The proposition characterizes when the platform wants to make it easier to rate. It captures the common finding that platforms want to balance surplus between buyers and sellers to generate value on the platform (Armstrong, 2006; Caillaud & Jullien, 2003; Rochet & Tirole, 2003). Combining this result with Proposition 3, we find that when $\pi_s - t < u_b$, the platform encourages ratings to raise $\pi_s - t$. Intuitively, a lower e raises profits per seller but also lowers the number of buyers who join the platform. $\pi_s - t < u_b$ implies that the former effect dominates. Thus, the platform facilitates ratings to lower harvesting rents and shift surplus to sellers, but thereby undermines the informativeness of ratings.

These results support concerns of regulators that platforms design insufficiently informative ratings environments for consumers, and that minimum standards of ratings may help to protect consumers (Competition and Markets Authority (UK), 2017).

We made the simplifying assumptions that sellers are ex-ante identical and do not yet know their relative quality level when entering the platform. Corollary 2 suggests that relaxing this assumption could lead to a novel selection effect. Platforms may still prefer to lower e in order to raise average seller surplus. But this discourages high-quality firms from joining the platform and leads to lower average quality on the platform. This way, facilitating ratings can undermine a key purpose of a ratings system.

These results suggest that a platform may change its strategy over time. For example, a new platform may choose a relatively large e to get informative ratings that favor buyers in order to grow and reinforce network effects, but later lower e to extract more profits from sellers.

[36] Documented by a website which guides non-mandarin speakers the use of Taobao (TaobaoTranslate, 2021).
[37] As explained by Amazon (Amazon, 2021).
[38] As is required by Steam ("Introducing Steam Reviews", 2021).

B.1.2 Competitive environment

Our basic model focuses on monopoly sellers. We now introduce competition to the model. To do so in a simple way, we assume that there exists a competitive fringe of non-strategic firms that offers a product of quality q at a price equal to their marginal cost c, where $q \geq 0$. Intuitively, we capture competition by changing the outside option of purchasing in the market to $q - c \geq 0$. This could capture (i) established firms for which consumers do not need ratings to evaluate the quality of their products; (ii) brick-and-mortar stores; or (iii) the expected utility a consumer gets when participating in another ratings environment such as another marketplace.

We now study how the presence of the competitive fringe affects the equilibrium strategy of our previous strategic sellers.

To start, suppose $q - c = 0$. Then the competitive fringe offers the same value as our previous outside option and the equilibrium is the same as in Proposition 1. Now suppose c decreases marginally and the competitive fringe offers a small but strictly positive surplus $q - c > 0$. This encourages sellers to harvest more ratings and reduces the informativeness of ratings. Intuitively, in period 1 firms that charge $\overline{p}$ extract consumers' conditional expected value. As c decreases, firms can no longer extract all surplus and reduce $\overline{p}$ to remain more beneficial to consumers than the fringe. The low-quality firms that charge $\underline{p}$, however, already leave a rent to consumers, which is why $\underline{p}$ does not decrease. Thus, the fringe puts more pressure on $\overline{p}$ than on $\underline{p}$, which is why low-quality firms harvest more ratings, leading to less-informative ratings in equilibrium (lower δ^*).

As c decreases further, $q - c$ becomes so large that it puts equal pressure on $\overline{p}$ and $\underline{p}$, so that the competitive fringe no longer affects the incentives to harvest ratings and the informativeness of ratings, δ^*.

The following proposition summarizes this result.

Proposition 7. $\frac{\partial \delta^*}{\partial c} > 0$ *if* $q^L - (q - c) \geq q^L - e$. *Otherwise,* $\frac{\partial \delta^*}{\partial c} = 0$

The proposition shows that competition puts disproportionate pressure on firms that extract more surplus from consumers, i.e. firms that do not harvest ratings. This is why competition encourages firms to harvest ratings and makes ratings less-informative. Crucially, however, ratings will not become uninformative: as the fringe becomes more competitive ($q - c$ increases), it will put equal pressure on all prices and no longer affect incentives to harvest ratings, leaving the informativeness of ratings constant at $\delta^* = \frac{1}{2}$.

Even though competition makes ratings less-informative, it unambiguously benefits consumers through the following two channels. First, competition exerts pressure on the higher price and lowers $\overline{p}$. Second, competition encourages low-quality firms to harvest ratings and charge $\underline{p}$, for which consumers receive a higher surplus. Thus, our results suggest that making consumers aware of alternative sellers can make ratings less-informative, but benefits consumers nonetheless.

B.1.3 Negative Ratings

For exposition we made the simplifying assumption of only good and no ratings. In this section, we relax this assumption to allow for more than 2 ratings, allowing consumers to rate positively, negatively, or not at all. In doing so, we show that if consumers rate value-for-money, only extreme ratings are played in equilibrium.

Formally, we setup a model that is similar to the base model. First, we modify the ratings available, such that $R_t \in \{-1, 0, 1\}$, representing bad, no, and good ratings respectively. Second, we relax the assumption that $\kappa = 1$ and $\Delta = 1$. This results in a more general function form of rating utility (see footnote 12 of the main text). The rating utility of consumers in period t is $v_t = [\kappa q^j - p_t^j]\Delta - e$, where $\kappa \in [0, 1]$ and $\Delta \in \{-1, 1\}$.[39] Here, $\Delta = 1$ represents a positive form glow from leaving a good rating, while $\Delta = -1$ represents a negative warm glow when leaving a bad rating.

In this setting, we show that there exist a range of prices for which each rating is possible: (i) when consumers receive a sufficiently high value-for-money, they choose to leave a good rating; (ii) when consumers receive a sufficiently low value-for-money, they choose to leave a bad rating; (iii) and for middle levels of value-for-money, consumers choose not to leave a rating.

However, there exist equilibria where low-quality firms choose between setting the prices of $\overline{p}$ and $\underline{p}$. This is because with Selection Assumption 1, we search for equilibria where consumers' beliefs are such that both $R_t \in \{0, -1\}$ are a reflection of a low-quality firm. This means that, conditional on receiving no or a bad rating, firms receive the same pay off in the second period. Hence, a profit maximising firm sets the highest possible price of $\overline{p}$ if it chooses not to receive a good rating.

Proposition 8 summarises the implication of this.

Proposition 8. *There exists a range of prices for which good, no and bad ratings may occur. In equilibrium, it is always beneficial for low-quality firms to obtain a bad rating over no rating, if consumers continue to buy at the price that induces a bad rating.*

[39]We specify $\Delta \in \{-1, 1\}$ for simplicity, and can be generalised to $\Delta \in \mathbb{R}$.

For completeness, we conduct a minor extension to show that our model connects well with suggestions in the empirical literature that consumers find it more difficult to leave a bad rating than they do a good rating (Cabral & Hortaçsu, 2010; Dellarocas & Wood, 2008; Filippas & Horton, 2022; Filippas et al., 2022). To do so, we modify the model with negative ratings to study the situation where consumers face different costs to leaving good and bad ratings. We say that consumers face a cost of e_b when leaving a bad rating, and a cost of e_g when leaving a good rating, where $e_b > e_g$.

Lemma 4. *When it is more difficult to leave a bad rating than a good rating, firms are more likely to obtain good ratings, and less likely to obtain bad ratings.*

Lemma 4 shows that when e_b is larger, firms are able to set a much higher price before they obtain a bad rating. This result implies that when e_b is sufficiently large, low-quality firms may not receive any bad ratings in equilibrium. Moreover, because e_g is relatively smaller, combined with Corollary 1, low-quality firms are likely to participate in ratings harvesting and receive a good rating.

We show that extreme ratings by the low-quality firm is an equilibrium result in a model of more than 2 ratings, and our model does not lose any information by considering a binary ratings system. Further, when considering the difference in cost needed to provide a rating, we provide theoretical foundation for the J-shape distribution of ratings found in the empirical literature, suggesting that it is an equilibrium result of value-for-money based ratings (Dellarocas & Wood, 2008; Filippas et al., 2022; Hu et al., 2009).

B.1.4 Continuum of quality

For exposition, we discussed a binary setting of firms with only high or low quality. In this section, We show that our results are robust to firms with a continuum of quality types.

Formally, we setup a model that is identical to the base model, with the following modifications: firm's quality is uniformly and continuously distributed on the interval $[0, 1]$; for simplicity there are no ratings prior to period 1 and consumers do not leave a rating in period 2; our solution concept is a pure-strategy perfect Bayesian equilibrium.

We show there exists an equilibrium where firms can be divided into 3 groups: (i) higher quality firms which set the highest possible price and receive a good rating in equilibrium; (ii) middle quality firms which choose to set lower prices, depending on their quality, to participate in ratings harvesting; (iii) lower quality firms that choose to set the highest possible price and forgo ratings.

Proposition 9. *There exists an equilibrium where: Firms with quality* $q \in [\hat{q}, 1]$ *obtain a good rating in equilibrium. Firms with quality* $q \in [\hat{\hat{q}}, \hat{q})$ *choose to participate in ratings harvesting in equilibrium. Firms with quality* $q \in [0, \hat{\hat{q}})$ *do not obtain a rating in equilibrium.*

In equilibrium, firms of low-quality obtain no rating and high- and middle- quality a good rating. Our result that easier ratings lead to less-informative ratings is robust: as it becomes easier to rate, firms of more types receive a good rating, reducing the expected quality of firms with a good rating. As in the main text, this occurs because as e decreases, firms harvest more ratings, causing $\hat{\hat{q}}$ to fall. This suggests that ratings become less informative because a good rating represents a wider range of quality types. However, despite ratings becoming less informative, the lowest quality firms still receive a good rating, suggesting that ratings are probably still somewhat useful for signalling quality.

B.1.5 Multi-period model

We check for robustness to time horizon by looking at a three period model. In this model, we find that equilibria similar to those described in our main model exist.

Notice that in a three period model, ratings take the following possible histories:

- $R_0 = \{0\}$
- $R_1 = \{01, 00\}$
- $R_2 = \{011, 010, 001, 000\}$

To save on notation, we omit the 0 in the initial period, capturing that all sellers start with the same reputation. We also extend Selection Assumption 1 to reflect the additional period.

Selection Assumption 3. *We focus on the equilibria where high-quality firms obtain a rating of* $R_t = 1$, $t \in \{1, 2\}$ *with probability 1.*

Further, in each period low-quality firms may decide to randomize with different probabilities, δ_t. Aside from the addition of a third period, the model remains the same as that of the main body of this paper.

Our intention is to uncover equilibrium strategies similar to the main body of the paper and not to consider all possible equilibria. We show that our findings continue to exist in a 3 period model. That is, we show that low-quality firms may choose to play a mixed strategy in every period. The result differs from our base setting in that a stricter set of restrictions is required for a unique mixed strategy to exist in every period.

We show that there exist equilibria where the low-quality firm chooses to play a mixed strategy in period 1 and in period 2 play a pure strategy: they harvest ratings if they received a good rating in the past or charge q^L otherwise. This means that low-quality firms choose to obtain good ratings in the first period even if they are unable to sustain this ratings outcome. As a result, we believe that our results are not driven by the end game effect of the final period. Instead, this effect is continuous and trading off pay offs in earlier periods for a big pay off in latter periods is a consideration made independent of the terminal period.

This is summarised in the following proposition

Proposition 10. *A mixed strategy equilibrium with properties similar to the base model exists in a 3-period model.*

Therefore, we conclude that the main results of our paper are robust to multiple periods and limiting our analysis to 2 periods in the main text helps to focus the discussion on the direct implications of value-for-money based ratings.

B.2 Proofs for Extensions Robustness (Intended For Online Appendix)

Proof of Proposition 6

Proof of Proposition 6.
To proof Proposition 6, we show that $\frac{\partial \pi_p}{\partial e} < 0$.

To begin we characterize the actions of users on either side of the platform. Afterwards, we look at the strategy of the profit-maximizing platform.

To begin we characterize when consumers join the platform. The consumers' value from joining the platform is $n_s u_b$, i.e. the consumer gets expected surplus u_b from each interaction with a seller, and there are n_s sellers in total. Consumers' outside option is uniformly distributed on $[0, 1]$, which is why buyer demand is given by $n_b^* = n_s^* u_b$.

Next, we consider the firms. Since firms are ex-ante homogeneous, they have the same ex-ante expected revenue per transaction, π_s. Firms also face the same commission fee, r, which is set by the platform. Additionally, all firms face a marginal cost t of selling on the platform. Therefore the per-transaction profit for firms is $\pi_s(1 - r) - t$. Since all firms face the same cost of entry $\bar{v}_s$, then any firm whose total profits are weakly above the outside option joins the platform. This means that $n_b^*(\pi_s(1 - r) - t) \geq \bar{v}_s$. Because all firms face the same decision, the number of firms to join the platform is either $n_s^* = 0$ or $n_s^* = 1$. If $n_s^* = 0$, there is no activity on the platform and the platform earns zero profits, which is not optimal. We conclude that $n_s^* = 1$. Note that this implies $n_b^* = u_b$.

We now turn our attention to the platform. Because sellers are homogeneous and $n_s^* = 1$, the profit-maximizing platform extracts the highest possible benefit from the royalty fee subject to sellers participating. This implies that the platform sets the optimal r^* such that

$$n_b^*(\pi_s(1-r^*)-t) = \bar{v}_s \Leftrightarrow r^* = 1 - \frac{\bar{v}_s}{n_b^*\pi_s} - \frac{t}{\pi_s}.$$

Using this, we can simplify the platform's profits to

$$\pi_p = n_b^*\pi_s r^* = n_b^*\pi_s - \bar{v}_s - tn_b^* = n_b^*(\pi_s - t) - \bar{v}_s.$$

Now, we consider the effects of the ratings environment on the profits of the platform. To see this, we need to understand how the platform's profits are affected by changes to effort cost,

$$\frac{\partial \pi_p}{\partial e} = \frac{\partial u_b}{\partial e}(\pi_s - t) + \frac{\partial \pi_s}{\partial e}u_b. \tag{14}$$

To understand the platform's strategy, we evaluate $\frac{\partial \pi_p}{\partial e}$. To do so, we first show that $\frac{\partial u_b}{\partial e} = -\frac{\partial \pi_s}{\partial e}$.

Consider first u_b. This is the transaction benefit of each consumer. Because consumers purchase first or second with equal probability, their ex-ante expected benefit per transaction is $u_b = \frac{1}{2}CS$, where we know from (2) that $CS = (1-\delta^*)(1-\gamma)e$. Thus,

$$\frac{\partial u_b}{\partial e} = \frac{1}{2}\frac{(1-\gamma)(1-\delta^*)}{\Delta} - \frac{1}{2}\frac{\partial \delta^*}{\partial e}e.$$

Next, consider π_s. This is the per transaction profit of firms before taking into account the commission fee of the platform. Ex-ante, this is equivalent to the expected revenue that the firms receives per consumer, i.e.

$$\begin{aligned}
\pi_s =& \frac{1}{2}\gamma[\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + \\
& \frac{1}{2}(1-\gamma)[(1-\delta^*)[q^L - e + \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)}] + \delta^*[\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L]] \\
=& [\gamma q^H + (1-\gamma)q^L] - \frac{1}{2}(1-\delta^*)(1-\gamma)e
\end{aligned}$$

Taking the derivative,

$$\frac{\partial \pi_s}{\partial e} = \frac{1}{2}\frac{\partial \delta^*}{\partial e}e - \frac{1}{2}(1-\gamma)(1-\delta^*).$$

Thus we conclude that $\frac{\partial u_b}{\partial e} = -\frac{\partial \pi_s}{\partial e}$.

Returning to equation (14), if $\frac{\partial \pi_p}{\partial e} < 0$, the platform designs a ratings system such that it minimizes the effort cost associated with rating.

$$\frac{\partial \pi_p}{\partial e} < 0 \Leftrightarrow \frac{\partial u_b}{\partial e}(\pi_s - t) + \frac{\partial \pi_s}{\partial e}u_b < 0 \Leftrightarrow \frac{\frac{\partial u_b}{\partial e}}{u_b} < -\frac{\frac{\partial \pi_s}{\partial e}}{(\pi_s - t)}.$$

From this formulation, we see that the platform's design decision depends on the relative elasticity of consumer transaction surplus and firms transaction revenue. In particular, since $\frac{\partial u_b}{\partial e} = -\frac{\partial \pi_s}{\partial e}$,

$$\frac{\frac{\partial u_b}{\partial e}}{u_b} < \frac{\frac{\partial u_b}{\partial e}}{(\pi_s - t)} \iff (\pi_s - t) < u_b$$

implies that $\frac{\partial \pi_p}{\partial e} < 0$.

We now show when this condition can be satisfied. First, note that $e < e^{cs}$, so $\frac{\partial u_b}{\partial e} > 0$.

We now provide the conditions for which $\frac{\frac{\partial u_b}{\partial e}}{u_b} < \frac{\frac{\partial u_b}{\partial e}}{(\pi_s - t)}$ is satisfied.

$$\pi_s - t < u_b \Leftrightarrow \frac{\gamma q^H + (1-\gamma)q^L - t}{e} < (1-\delta^*)(1-\gamma). \tag{15}$$

Therefore, when (15) holds, we show that platforms favour sellers and minimise the effort cost required to leave a rating.

We conclude that when (15) holds platforms will minimize the effort costs required to leave a rating.

Specifically, when $t \geq \frac{2q^L\sqrt{4\gamma^2(q^H-q^L)^2+(1+\gamma)^2}}{2}$, $e^* = 0$. Otherwise,

$$e = \frac{(t-q^L)(1-\gamma) \pm \sqrt{(q^L-t)^2(1+2\gamma+\gamma^2) - 4\gamma^3(q^H-q^L)^2}}{2\gamma}$$

Since $e^* > 0$, we reject the negative and

$$e^* = \frac{(t-q^L)(1-\gamma) + \sqrt{(q^L-t)^2(1+2\gamma+\gamma^2) - 4\gamma^3(q^H-q^L)^2}}{2\gamma}$$

We have provided the condition for which platforms are incentivised to minimize effort costs associated with rating. And we have also shown more generally the level of effort that maximises platform's profit when (15) holds.

This concludes the proof. $\square$

Proof of Proposition 7

To proof Proposition 7, we first need to show some lemmas. First, Lemma 5 guides when the competitive fringe may be considered to be active in the market. Second, Lemma 6 shows the adjusted pricing strategies of strategic firms. Finally, Lemma 7 characterizes the adjusted consumer's beliefs in the presence of the competitive fringe. We then use these results to prove Proposition 7.

Lemma 5. *The competitive fringe is only relevant whenever $c < q$.*

Proof of Lemma 5.
Whenever $c > q$, consumers receive negative utility from the competitive fringe. Whenever $c = q$, consumers receive 0 utility. Since the strategic firms provide at least non-negative utility, by assumption they would receive the full consumer demand. Therefore, the fringe only captures consumers if $c < q$.

This concludes the proof. $\square$

Lemma 6. *Pricing strategy of the high-quality firm:*

- *Period 1:* $p_1'^H = E[q_1|R_0, p_1^H] - (q - c)$
- *Period 2:* $p_2'^H = E[q_2|R_1] - (q - c)$

Pricing strategy of the low-quality firm:

- *Period 1:*
  - $R_1 = 1$, $p_{1,1}'^L = min\{q^L - (q - c), q^L - e\}$
  - $R_1 = 0$, $p_{1,0}'^L = p_1'^H$
- *Period 2:* $p_2'^L = E[q_2|R_1] - (q - c)$

Proof of Lemma 6.
To prove this, we first discuss the tie breaker rule. Second, we consider the second period as

this is straight forward. Then we consider the high-quality firm in the first period, followed by the low-quality firm in the first period. The proofs are similar to that of Lemma 2.

Our tie breaker rule is such that whenever both the strategic firm and the fringe provide the same level of consumer surplus, consumers choose to purchase from the strategic firm. Here, we shall argue why this tie breaker rule is a simplification that yields virtually identical results to a tie breaker rule where consumers randomise between the two firms.

By construction, the fringe sets some fixed price c, while the strategic firm selects prices. Therefore, for any situation where the strategic firm and fringe provide the same level of consumer surplus, the strategic firm can always choose to set prices some small $\epsilon > 0$ lower, such that they obtain the full demand. Therefore, by assuming this tie breaker rule, we are able to simplify our discussion without considering the need for some small ϵ deviation.

We now turn our attention to the discussion of equilibrium prices in the second period. In the second period, consumers are aware that consumption from the fringe will leave them a surplus of $q-c$. This forms the outside option for consumers. Strategic firms therefore have to provide consumers with a surplus of at least $q-c$, doing so will shift all the demand towards the strategic firm. We conclude that the firm will not set prices lower than $E[q_2|R_1]-(q^L-c)$.

We now turn our attention to the high-quality firm in the first period. As in Lemma 2, a high-quality firm wishing to obtain a positive rating must set prices no higher than $q^H - e$. As in the proof of Proposition 5, consumers are aware that consumption of the outside good will provide $q - c$ level of utility. Therefore, firms must set prices no higher than $E[q_1|R_0, p_1] - (q - c)$. Because we show in the proof of Proposition 5 that we must have $min\{q^H - e, E[q_1|R_0, p_1^H]\} = E[q_1|R_0, p_1^H]$ in equilibrium, high-quality firms will continue to receive a good rating at $E[q_1|R_0, p_1] - (q - c)$. It follows that the equilibrium price is $p_1'^H = E[q_1|R_0, p_1^H] - (q - c)$.

We now look at the low-quality firm in period 1. When the low-quality firm prefers to obtain a good rating, it has to set a price no higher than $q^L - e$. With the presence of the competitive fringe, consumers are aware that they would be able to receive at least $q - c$ of surplus. Therefore, in order for the low-quality firm to capture demand and get a good rating, they are unable to command prices higher than $q^L - (q - c)$. Thus, the equilibrium price is $p_{1,1}'^L = min\{c, q^L - e\}$.

For the low-quality firm receiving no rating, it sets prices above q^L-e. The profit maximizing firm will set the highest possible price at which consumers buy. For the same argument as in the proof of Lemma 2, low-type firms do not set prices higher than the high-quality firm. Therefore, when low-quality firms receive no rating, it sets prices equal to that of the high-quality firm.

This concludes the proof. $\square$

Lemma 7. *In the first period, consumer's beliefs for each equilibrium price p_1 is given by*

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} & \text{if } p_1 > \overline{p_1^L} \\ q^L & \text{if } p_1 \leq \overline{p_1^L}, \end{cases}$$

and in the second period,

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma)q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0 \end{cases}$$

Proof of Lemma 7.
The proof is identical to that of Lemma 3. $\square$

Proof of Proposition 7.
We now proof Proposition 7.

First, we discuss the possible cases for how the competitive fringe might affect the behavior of strategic firms. Then we consider these cases one at a time and find the effect that changes in c have on the mixed-strategy of the low-quality firm.

Since we know that the second period prices are equally affected by the competitive fringe in all cases, we turn our attention to the first period. Notice that in period 1, the high-quality firm sets a single price, and the low-quality firm sets two out of three possible prices, i.e. $q^L-(q-c)$, q^L-e, or $p_1'^H$. Further, notice that $\overline{p_1^H} > \overline{p_1^L}$ and $E[q_1|p_1'^H]-(q-c) > q^L-(q-c)$. We know from the proof of Proposition 5 that in equilibrium we have $\overline{p_1^H} > E[q_1|p_1'^H]$, this leaves us with the following possible scenarios:

$$\begin{aligned} &1. \overline{p_1^H} > E[q_1|p_1'^H] - (q-c) > \overline{p_1^L} > q^L - (q-c) \\ &2. \overline{p_1^H} > E[q_1|p_1'^H] - (q-c) > q^L - (q-c) \geq \overline{p_1^L} \end{aligned}$$

We consider the scenarios individually.

1. $\overline{p_1^H} > E[q_1|p_1'^H] - (q-c) > \overline{p_1^L} > q^L - (q-c)$

Because $\overline{p_1^L} = q^L - e > q^L - (q-c)$, we know from Lemma 6 that the low-quality firm is indifferent between setting prices $p_1'^H$ and $q^L - (q-c)$. When charging $p_1'^H$, the firm gets no rating and charges $q^L-(q-c)$ in period 2. When charging $q^L-(q-c)$, the firm gets a good rating and earns the expected quality of a firm with a good rating in period 2 minus $(q-c)$. This leads to the following condition.

$$
\begin{aligned}
& E[q_1|p_1'^H]-(q-c)+q^L-(q-c)=q^L-(q-c)+E[q_2|R_1]-(q-c)\\
\Leftrightarrow & \frac{\gamma q^H+\delta^*(1-\gamma)q^L}{\gamma+\delta^*(1-\gamma)}=\frac{\gamma q^H+(1-\delta^*)(1-\gamma)q^L}{\gamma+(1-\delta^*)(1-\gamma)}\\
\Leftrightarrow & \delta^*=0.5,
\end{aligned}
$$

which is independent of c.

This concludes case 1.

2. $\overline{p_1^H}>E[q_1|p_1'^H]-(q-c)>q^L-(q-c)\geq\overline{p_1^L}$

Because $q^L-(q-c)\geq\overline{p_1^L}$, and we know from Lemma 6 that the low-quality firm is indifferent between setting prices $p_1'^H$ and $\overline{p_1^L}=q^L-e$. To be indifferent between these prices, the following condition must hold. The left-hand side is as in the previous case. On the right-hand side, the firm charges q^L-e in period 1 and obtains a good rating. In period 2, it earns the expected quality of a firm with good rating in period 2 minus $(q-c)$. This leads to the following condition.

$$
\begin{aligned}
& E[q_1|p_1'^H]-(q-c)+q^L-(q-c)=\overline{p_1^L}+E[q_2|R_1]-(q-c)\\
\Leftrightarrow & \frac{\gamma q^H+\delta^*(1-\gamma)q^L}{\gamma+\delta^*(1-\gamma)}-q+c=q^L-e+\frac{\gamma(q^H-q^L)}{\gamma+(1-\delta^*)(1-\gamma)}.
\end{aligned}
$$

Using the implicit-function theorem then leads to

$$
\frac{\partial\delta^*}{\partial c}=\frac{(\gamma+(1-\delta^*)(1-\gamma))^2(\gamma+\delta^*(1-\gamma))^2}{\gamma(1-\gamma)(q^H-q^L)[(\gamma+(1-\delta^*)(1-\gamma))^2+(\gamma+\delta^*(1-\gamma))^2]}>0.
$$

This concludes case 2.

We conclude that $\frac{\partial\delta^*}{\partial c}>0$ if $q^L-(q-c)\geq q^L-e$ and 0 otherwise. $\square$

Proof of Proposition 8

We can proof this proposition using the following two Lemmas

Lemma 8. *Consumers obtain the same signal from a negative rating and no rating.*

Proof of Lemma 8.

This proof holds directly from Selection Assumption 1. Consumers' belief is such that high-quality firms always get a good rating. Hence, on observing no or bad ratings, they would believe this to be obtained by low-quality firms.

$$E[q_2|R_1] = \begin{cases} \frac{\gamma q^H + (1-\delta^*)(1-\gamma) q^L}{\gamma + (1-\delta^*)(1-\gamma)} & \text{if } R_1 = 1 \\ q^L & \text{if } R_1 = 0 \\ q^L & \text{if } R_1 = -1 \end{cases}$$

where δ^* continues to represent the probability that low-quality firms get no rating.

This concludes the proof. $\square$

Lemma 9. *When low-quality firms choose not to receive a good rating, it prefers to receive a bad rating over no rating, if consumers continue to buy at the price level that induces a bad rating.*

Proof of Lemma 9.
First, we discuss the prices set in the second period. Then we look at consumer's beliefs in the first period and the prices set in the first period. We then show that in equilibrium, there exist some price for which low-quality firms receive no rating, and another price where they receive bad ratings.

When looking at the prices set in the second period, notice first that firms of all types will set the highest possible price in the second period. This is equivalent to consumer's expectation in the second period. Since this is only dependent on ratings, on receiving a good rating in the first period, firms set $\frac{\gamma q^H + (1-\delta^*)(1-\gamma) q^L}{\gamma + (1-\delta^*)(1-\gamma)}$; on receiving no rating, they set a price of q^L; on receiving a bad rating, they also set a price of q^L.

Now, consider the possible prices in the first period. In the first period, high-quality firms will set a price $p_1^H = min\{\kappa q^H - \frac{e}{\Delta}, E[q_1|R_0, p_1^H]\}$ and receives a good rating. The proof of this is identical to the proof of high-quality firm's prices in Lemma 2.

Low-quality firm's do one of the following: set a price such that it receives a good rating, no rating or a bad rating.

When a low-quality firm receives a good rating, it sets a unique price and this is $p_{1,1}^L = \kappa q^L - \frac{e}{\Delta}$. This proof is identical to that of Lemma 2 when low-quality firms receive a good rating.

We turn our attention to the situation when low-quality firms receive a bad rating. When a low-quality firm receives a bad rating, this only occurs if $[\kappa q^L - p_1^L]\Delta - e \geq 0$. A bad rating means consumers exhibit a negative warm glow, and $\Delta = -1$. Therefore, consumers leave a bad rating whenever $p_1^L \geq \kappa q^L + \frac{e}{\Delta}$, gaining a positive rating utility from doing so. From Selection Assumption 2, we know that since this price is larger than $\overline{p_1^L}$, consumers expectations are fixed and from Lemma 2, we know that the firm sets the highest possible price. Therefore, when obtaining a bad rating, the low-quality firm sets the price $p_1^L = E[q_1|R_0, p_1^H]$.

Now we turn our attention to the situation when low-quality firms receive no rating. This occurs when $[\kappa q^L - p_1^L]\Delta - e < 0$. That is the rating utility from leaving a good or bad rating is negative. Recall that the utility from giving no rating is 0.

Suppose that the consumer considers between leaving a good rating and no rating. Since consumers receive a positive warm glow from a good rating, this implies that $\Delta = 1$. Therefore, no rating only occurs if $[\kappa q^L - p_1^L] < \frac{e}{\Delta}$. In other words, $p_1^L > \kappa q^L - \frac{e}{\Delta}$.

Now suppose that the consumer considers between leaving a bad rating and no rating. Since consumers receive a negative warm glow from a bad rating, this implies that $\Delta = -1$. Therefore, no rating only occurs if $p_1^L < \kappa q^L + \frac{e}{\Delta}$ and $p_1^L > \kappa q^L - \frac{e}{\Delta}$ hold together.

Notice that $\kappa q^L - \frac{e}{\Delta} < \kappa q^L + \frac{e}{\Delta}$. This implies that there exist a range of prices, $p_1^L \in (\kappa q^L - \frac{e}{\Delta}, \kappa q^L + \frac{e}{\Delta})$ such that consumers maximising their utility provide no ratings to low-quality firms.

In equilibrium, the total profit of a low-quality firm obtaining a bad rating is $\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)} + q^L$, while the total profit of obtaining no rating is strictly below $\kappa q^L + \frac{e}{\Delta} + q^L$.

We can now show that low-quality firms set some unique price in equilibrium, when they choose not to obtain a good rating. In equilibrium, when consumers observe price of p_1^H, they anticipate a utility of $\frac{\gamma q^H + \delta^*(1-\gamma)q^L}{\gamma + \delta^*(1-\gamma)}$. If this is above $\kappa q^L + \frac{e}{\Delta}$, low-quality firms maximise their profits by extracting the full surplus from consumers, and obtaining a bad rating. Alternatively, if the anticipated utility is below $\kappa q^L + \frac{e}{\Delta}$, it is not profitable for firms to set $\kappa q^L + \frac{e}{\Delta}$ as this would lead to zero demand. Instead, the low-quality firm sets p_1^H and obtains no rating. Therefore, when choosing not to obtain a good rating, low-quality firms always sets the unique price mimicking price of p_1^H.

We conclude that when we allow for negative ratings, there exists a range of prices for which good, bad, and no ratings may occur. However, in equilibrium, a profit maximising low-quality firm mixes between setting a price that allows it to harvest ratings and price mimicking. This allows it to obtain the extreme ratings of no, or negative rating depending on the cost of leaving a rating.

This concludes the proof. $\square$

Together, Lemma 8 and Lemma 9 show that in the equilibrium, negative ratings will replace no ratings when going from a system of 2 ratings options to 3 ratings options, and consumers continue to buy at the price level that induces a negative rating. This concludes the Proof of Proposition 8.

Proof of Lemma 4.
To complete this proof, first consider the possible prices played by the high-quality firm. This is unchanged and follows directly from the Proof of Lemma 9. Second, we now consider the possible prices played by the low-quality firm.

When receiving a good rating, the low-quality firm sets a unique price and this is given by $p^L_{1,1} = \overline{p^L_1} = \kappa q^L - \frac{e_g}{\Delta}$. This proof is identical to that of Lemma 2 when the low-quality firm receives a good rating.

Next, we turn our attention to the low-quality firm receiving a bad rating. This only occurs if $[\kappa q^L - p^L_1](-\Delta) - e_b > 0$. As in Lemma 9 consumers providing a bad rating receive a negative warm glow, and negative ratings occur if $p^L_1 > \kappa q^L + \frac{e_b}{\Delta}$.

Notice that applying the same logic as in Lemma B.1.3, there exists a range of prices such that no rating occurs, and this is $p^L_1 \in (\kappa q^L - \frac{e_g}{\Delta}, \kappa q^l + \frac{e_b}{\Delta})$.

This has two implications. First, as e_g is smaller, then from Corollary 1, low-quality firms harvest ratings more.

Second, as e_b is larger, low-quality firms only receive a bad rating if they set a higher price. However, as this runs up against the upper bound of prices that they may set, prices must be less than consumers willingness to pay, firms are unlikely to receive bad ratings.

This concludes the proof. $\square$

Proof of Proposition 9

Proof of Proposition 9.
To begin the proof, we first define the game. This is a two period game comprising of a firm active in both periods, and a different consumer in each period. Before the first period, the firm draws quality q from a uniform distribution on the interval $[0, 1]$, learning its true quality.

In the first period, firms choose any price p_1, $\sigma_1^f : q \in [0,1] \to p_1 \in \mathbb{R}$. First period consumers learn of this price, forming beliefs over the quality of the firm, $E[q|p_1] : p_1 \in \mathbb{R} \to [0,1]$. They then make a consumption decision, $\sigma_{1,B}^c : p_1 \in \mathbb{R} \to \{0,1\}$, where 1 represents the decision to purchase and 0 not. Upon consumption, and payment, consumers learn of the true quality of the product. They may then choose to leave a rating, $\sigma_{1,R}^c : q \times p_1 \in [0,1] \times \mathbb{R} \to R_1 \in \{0,1\}$, where 1 represents the decision to rate, and 0 not.

At the end of the first period, first-period consumers leave the market and second-period consumers arrive. The rating R_1 is revealed to both the firm and the second-period consumer. In the second period, the firm sets a price p_2, $\sigma_2^f : q \times R_1 \in [0,1] \times \{0,1\} \to p_2 \in \mathbb{R}$. Second-period consumers learn this price, forming beliefs over the quality of the firm, $E[q|p_2, R_1] : p_2 \times R_1 \in \mathbb{R} \times \{0,1\} \to [0,1]$. Consumers then make a consumption decision, $\sigma_2^c : p_2 \times R_1 \in \mathbb{R} \times \{0,1\} \to \{0,1\}$.

Candidate equilibrium: We now describe the candidate equilibrium: Firms with quality $q \in [\hat{q}, 1]$ obtain a good rating in equilibrium. Firms with quality $q \in [\hat{\hat{q}}, \hat{q})$ choose to participate in ratings harvesting in equilibrium. Firms with quality $q \in [0, \hat{\hat{q}})$ do not obtain a rating in equilibrium. The cutoffs are $\hat{q} = \frac{5}{8} + \frac{e}{2}$ and $\hat{\hat{q}} = \frac{1}{8} + \frac{e}{2}$.

We first state consumers' equilibrium beliefs in period 1.

$$E[q|p_1] = \begin{cases} \frac{\int_{\hat{q}}^1 q dq + \int_0^{\hat{\hat{q}}} q dq}{\int_{\hat{q}}^1 1 dq + \int_0^{\hat{\hat{q}}} 1 dq} = \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})} & \text{if } p_1 \geq \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})} \\ q & \text{if } p_1 = q - e < \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})} \\ 0 & \text{otherwise} \end{cases}$$

Note that $\hat{q}$ is such that $p_1(\hat{q}) = \hat{q} - e = \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$, and the expectation is well defined.

Next, we state consumer's strategies in period 1.

$$\sigma_{1,B}^c = \begin{cases} 1 & \text{if } p_1 \leq E[q|p_1] \\ 0 & \text{if } p_1 > E[q|p_1] \end{cases}$$

$$\sigma_{1,R}^c = \begin{cases} 1 & \text{if } p_1 \leq q - e, \ \ \forall q \\ 0 & \text{if } p_1 > q - e, \ \ \forall q \end{cases}$$

Next, we state the firms' strategies in period 1.

$$\sigma_1^f = \begin{cases} \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})} & \text{if } q \in [\hat{q}, 1] \\ q - e & \text{if } q \in [\hat{\hat{q}}, \hat{q}) \\ \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})} & \text{if } q \in [0, \hat{\hat{q}}) \end{cases}$$

We now turn our attention to period 2, stating consumers' equilibrium beliefs, their strategies, and the firms' strategies.

$$E[q|p_2, R_1] = \begin{cases} \frac{\int_{\hat{\hat{q}}}^1 q dq}{\int_{\hat{\hat{q}}}^1 1 dq} = \frac{1+\hat{\hat{q}}}{2} & \text{if } R_1 = 1, \forall p_2 \\ \frac{\int_0^{\hat{\hat{q}}} q dq}{\int_0^{\hat{\hat{q}}} 1 dq} = \frac{\hat{\hat{q}}}{2} & \text{if } R_1 = 0, \forall p_2 \end{cases}$$

$$\sigma_{2,B}^c = \begin{cases} 1 & \text{if } p_2 \leq E[q|p_2, R_1] \\ 0 & \text{if } p_2 > E[q|p_2, R_1] \end{cases}$$

$$\sigma_1^f = \begin{cases} \frac{\int_{\hat{\hat{q}}}^1 q dq}{\int_{\hat{\hat{q}}}^1 1 dq} & \text{if } R_1 = 1, \forall q \\ \frac{\int_0^{\hat{\hat{q}}} q dq}{\int_0^{\hat{\hat{q}}} 1 dq} & \text{if } R_1 = 0, \forall q \end{cases}$$

Note that $E[q|p_2, R_1 = 1] = E[q|p_2, R_1 = 0] + \frac{1}{2}$. This implies that obtaining a good rating allows firms to set a higher price in the second period.

Optimality: We now show that consumers make rational choices given specified beliefs. In the first period, given their information set, i.e. p_1, consumers only choose to consume if their expected utility is weakly positive. Moreover, they only choose to leave ratings if their utility from doing so, $v_t(q)$ is weakly positive. Period 2 consumers observe p_2 and R_1, making a consumption decision only if their expected utility conditional on observing p_2 and R_1 is weakly positive. Therefore, we conclude that the candidate equilibrium specifies optimal strategies for consumers given their beliefs.

We now show that firms make rational choices given specified beliefs. To do so, we first solve for $\hat{q}$ and $\hat{\hat{q}}$. Next, we show that there is no profitable deviation from the firm's specified strategy.

To solve for $\hat{q}$ and $\hat{\hat{q}}$, we use two conditions. First, consumers who observe $\hat{q}$ are indifferent between rating or not, i.e. $\hat{q} - e = \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$. Second, firm $\hat{\hat{q}}$ is indifferent between rating harvesting and price mimicking, that is $\hat{\hat{q}} - e + \frac{1}{2} = \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$. Combining these two equations, $\hat{q} - \hat{\hat{q}} = \frac{1}{2}$. This implies that half the firms always participate in ratings harvesting. And we have $\hat{q} = \frac{5}{8} + \frac{e}{2}$, $\hat{\hat{q}} = \frac{1}{8} + \frac{e}{2}$.

We now show that firms have no profitable deviation. In the second period, firms charge prices equal to their expected quality conditional on their rating and earns strictly positive profits. Thus, deviating to a larger price reduces demand to zero and is therefore not profitable. Deviating to a lower price reduces margins without increasing demand, and is therefore also not profitable. We conclude that firms have no profitable deviation in the second period.

Next, we show that firms have no profitable deviation in the first period. Note first that any deviation to a price $p_1 > \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$ induces zero demand and profits and is therefore not a profitable deviation.

Next, consider firms of quality $q \in [\hat{q}, 1]$. These firms get a rating and therefore charge the large price in period 2. We have shown above that charging a larger price above $\frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$ is not a profitable deviation. Additionally, deviating to a strictly lower price only reduces margins in period 1 without increasing demand in period 1 or changing the rating in period 2. Thus, deviating to a lower price in period 1 is not a profitable deviation. We conclude that firms of $q \in [\hat{q}, 1]$ do not have a profitable deviation in period 1.

Next, consider firms of quality $q \in [\hat{\hat{q}}, \hat{q})$. In the candidate equilibrium, they set a price $p_1 = q - e$ and receive a good rating. To start, consider an upward deviation. We have shown above that the firm does not want to deviate to a price strictly above $\frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$. Because any deviation to a price above $q - e$ leads consumers to no longer rate the firm, the most profitable deviation to a larger price is therefore to the price $\frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$, i.e. the firm would deviate to price mimicking. Because $\hat{q}$ is such that $\hat{q} - e = \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$, and because $q < \hat{q}$, this is indeed a deviation to a larger price. But because $\hat{\hat{q}}$ is such that $\hat{\hat{q}}$ is indifferent between rating harvesting and price mimicking, i.e. $\hat{\hat{q}} - e + \frac{1}{2} = \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{1-\hat{q}+\hat{\hat{q}}}$, and because $q - e + \frac{1}{2} > \hat{\hat{q}} - e + \frac{1}{2}$, firms $q \in [\hat{\hat{q}}, \hat{q})$ earn strictly larger profits from rating harvesting than price mimicking. Thus, firms $q \in [\hat{\hat{q}}, \hat{q})$ do not have a profitable deviation to a larger price. Additionally, if any firm $q \in [\hat{\hat{q}}, \hat{q})$ deviates to a strictly lower first-period price that $q - e$, the firm earns a strictly lower margin in period 1 without earning a larger demand in period 1 or a larger profit in period 2. We therefore conclude that no firm $q \in [\hat{\hat{q}}, \hat{q})$ has a profitable deviation in period 1.

Next, consider firms $q \in [0, \hat{\hat{q}})$, recall that in equilibrium they set a first-period price $\frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$ and receive no rating. We have therefore shown above that they have no profitable upward deviation. To see that there is no downward deviation, notice that the firms currently receive no rating. If they lower prices slightly, above $q - e$, they will still receive no rating, thus period 2 profits are unaffected and period 1 profits decrease, which does not constitute a profitable deviation. However, if they reduce prices to weakly below $q - e$, then these firms receive a good rating, and gain a profit of $\frac{1}{2}$ in the second period. But since $q < \hat{\hat{q}}$, we know that $q - e + \frac{1}{2} < \hat{\hat{q}} - e + \frac{1}{2} = \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$ and these deviations are also not profitable. We conclude that no firm $q \in [0, \hat{\hat{q}})$ has a profitable deviation in period 1.

Overall, we conclude that not firm has a profitable deviation and that the firms' strategies in the candidate equilibrium are optimal given their beliefs.

Correct beliefs on the equilibrium path: We now show that the beliefs in the candidate equilibrium follow Bayes rule on the path of play.

In the second period, consumers beliefs are such that $E[q|p_2, R_1] = E[q|R_1]$. Consumers correctly believe that only firms of quality $q \geq \hat{\hat{q}}$ receive a good rating in period 1. Similarly, consumers correctly believe that firms of quality $q < \hat{\hat{q}}$. Therefore their beliefs are consistent with Bayes rule.

In the first period, consumer beliefs $E[q|p_1]$ only depend on the first-period price. Consumers who observe a price $p_1 = \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$ correctly take into account that only firms $q \in [\hat{q}, 1]$ and $q \in [0, \hat{\hat{q}})$ set this price on the path of play, and these beliefs are correct. Next, note that the remaining firms $q \in [\hat{\hat{q}}, \hat{q})$ charge a price $p_1 = q - e < \frac{1-\hat{q}^2+\hat{\hat{q}}^2}{2(1-\hat{q}+\hat{\hat{q}})}$. Thus, each of these firms charges a unique price that no other firm charges in equilibrium, and consumers have correct beliefs. We therefore conclude that consumers' beliefs are consistent with Bayes rule on the path of play. Since firms have full information, also their beliefs are consistent with Bayes rule. We conclude that all players' beliefs are consistent with Bayes rule on the path of play.

Finally, we show that there indeed exist parameter ranges for which $\hat{q} \in (0, 1)$ and $\hat{\hat{q}} \in (0, 1)$. Because $\hat{\hat{q}} < \hat{q}$, it suffices to show that $0 < \hat{\hat{q}}$ and $\hat{q} < 1$. Reformulating $\hat{\hat{q}}$ to $\hat{\hat{q}} = \frac{1}{8} + \frac{e}{2}$, implies $0 < \hat{\hat{q}}$. And $\hat{q} = \frac{5}{8} + \frac{e}{2}$ implies $\hat{q} \leq 1 \iff e \leq \frac{3}{4}$. Therefore, we conclude that the equilibrium does indeed exist.

A larger e makes ratings more informative. Finally, we can show that the informativeness of ratings increases in e. Note that when e increases, more firms participate in price mimicking, $\frac{\partial \hat{\hat{q}}}{\partial e} = \frac{1}{2} > 0$. This implies that conditional on observing a good rating, consumers believe that the rating comes from firms of higher quality, as the cut off for firms receiving a good rating has increased. Thus ratings are more informative of quality.

We have shown that there exists a pure-strategy perfect-Bayesian equilibrium with 3 groups of firms, $q \in [\hat{q}, 1]$ obtaining a good rating, without influencing ratings, $q \in [\hat{\hat{q}}, \hat{q})$ choosing to participate in ratings harvesting, and $q \in [0, \hat{\hat{q}})$ which do not obtain a rating in equilibrium.

And also in this equilibrium, the informativeness of ratings increases in e.

$\square$

Proof of Proposition 10

To show Proposition 10, we prove a more general proposition, Proposition 11. Further, recall that to save on notation we omit the 0 rising from $t = 0$.

Proposition 11. *All perfect Bayesian equilibria satisfy the following.*

1. *High-quality firms receive a good rating with probability 1 and charge* $p_t^H = E[q_t|R_{t-1}, p_t^H]$, $t \in \{1, 2\}$.
2. *Low-quality firms randomize their strategy in period* $t \in \{1, 2\}$.
   a. *They charge* $\overline{p_t^L} = q^L - e$ *and obtain a good rating with probability* $1 - \delta_t^*$.
   b. *They charge* p_t^H *and obtain no rating with probability* δ_t^*.
3. *Firms set prices equal to expected quality conditional on ratings in the last period.*
4. *Consumer beliefs given equilibrium prices are given by:*
   a. *In period 1,* $E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} & \text{if } p_1 > \overline{p_1^L} \\ q^L & \text{if } p_1 \leq \overline{p_1^L} \end{cases}$
   b. *In period 2,*
$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} & \text{if } p_2 > \overline{p_2^L} \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination} \end{cases}$$
   c. *In period 3,* $E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2 \end{cases}$

Furthermore, we show that $\delta_1^* \in (0, 1)$ *and* $\delta_2^* \in (0, 1)$ *is an equilibrium if* $e < \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$, *and* $\delta_1^* \in (0, 1)$ *and* $\delta_2^* = 1$ *is an equilibrium if* $e \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$. *These equilibria exist if* $e < \frac{(1-\gamma)(q^H - q^L)}{2-\delta_2^*}$, $\gamma > \frac{1}{3}$, *and*
$q^H - e \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}\}$.

We proceed as follows. Before proving Proposition 11, we show two Lemmas. First, we use our Selection Assumptions to characterize firms' pricing strategies. Second, we pin down equilibrium beliefs. Finally, we use these results to show Proposition 11. Most of the arguments used in proof are similar to the ones from Proposition 1, which is why we briefly sketch them here.

We begin by pinning down the equilibrium prices in period 1 and 2 in Lemma 10.

Lemma 10. *In equilibrium, firms play the following prices in period 1 and 2 with weakly positive probability.*

- *High-quality firm:* $p_t^H = min\{q^H - e, E[q_t|R_{t-1}, p_t^H]\}$
- *Low-quality firm:*

$$p_{1,1}^L = p_{2,11}^L = q^L - e$$

$$p_{1,0}^L = p_{2,10}^L = E[q_t|R_{t-1}, p_t^H]$$

$$p_{2,00}^L = p_{2,01}^L = q^L$$

Proof of Lemma 10.
The proof of Lemma 10 is similar to the proof of Lemma 2.

The key difference lies in the possible rating histories. Given the extension to 3 periods, the possible rating histories are now $R_0 \in \{0\}, R_1 \in \{0, 1\}, R_2 \in \{00, 01, 10, 11\}$. As in the proof of Lemma 2, we will consider the pricing strategy of the high-quality firm followed by low-quality firm.

From Selection Assumption 1, a high-quality firm wants to set prices which allows it to get a good rating and therefore in equilibrium her rating's history are $R_0 = \{0\}, R_1 = \{1\}$ and $R_2 = \{11\}$. We show that on the equilibrium path $p_t^H = min\{q^H - e, E[q_t|R_{t-1}, p_t^H]\}$ for $t \in \{1, 2\}$.

To show that the high-quality firm sets a unique price in each period $t \in \{1, 2\}$ with probability 1, we follow the proof set out in Lemma 2. Suppose towards a contradiction that the high-quality firm sets more than one price with positive probability. Without loss of generality, suppose that the high-quality firm sets a continuous distribution of prices in either period, i.e. it charges prices in some interval $p_t \in [p_t', p_t'']$ such that $p_t'' > p_t'$, and the high-quality firm receives a good rating with probability 1 for all $p_t \in [p_t', p_t'']$. Note that when the high-quality firm gets a rating for all $p_t \in [p_t', p_t'']$, consumers purchase products with probability 1. Notice that for any price $\hat{p}_t > p_t$ such that $\hat{p}_t \in [p_t', p_t'']$, we have $\pi_t^H(\hat{p}_t) > \pi_t^H(p_t)$. The reason is that both prices induce the same demand in period t, and the same rating and therefore the same continuation profits. Thus, the firm can strictly increase profits by shifting all probability mass from $[p_t', p_t'']$ to p_t''. This contradicts the assumption that the high-quality firm sets an uncountably infinite prices with positive probability. Similarly, the firm will not set countably finite or infinite prices that induce the same rating. Therefore, we conclude that the high-quality firm sets a unique price in each period t on the equilibrium path with probability 1. We denote this as p_t^H.

Next, we show that there exist an upper bound on prices, $\overline{p_t^j}$ for $j \in \{L, H\}$ such that j-quality firm receives a positive rating. In order for a firm to induce a positive rating, the rating utility must be positive. Therefore,

$$q^j - p_t^j - e \geq 0 \iff p_t^j \leq \overline{p_t^j} \equiv q^j - e$$

Finally, consider that prices are bound by consumer's beliefs, $E[q_t|R_{t-1}, p_t^H] < \overline{p_t^H}$. Under such scenarios, by Selection Assumption 1, high-quality firms prefer obtaining a good rating. This can only be achieved if consumers buy. Therefore, p_t^H is bound by $E[q_t|R_{t-1}, p_t^H]$.

We now show that $p_t^H = min\{q^H - e, E[q_t|R_{t-1}, p_t^H]\}$ with probability 1. As with Lemma 2, note that $\overline{p_t^H} > \overline{p_t^L}$ and $q^L > \overline{p_t^L}$. Thus, because for equilibrium expectations we have $E[q_t|R_{t-1}, p_t^H] > q^L$, the high-quality firm sets equilibrium prices strictly larger than $\overline{p_t^L}$. Applying Selection Assumption 2, consumers have the same beliefs for all prices strictly above $\overline{p_t^L}$ in each period t. Since $p_t^H > \overline{p_t^L}$, these beliefs are given by $E[q_t|R_{t-1}, p_t^H]$, the correct equilibrium beliefs. Further, since consumers have the same beliefs for all prices above $\overline{p_t^L}$ in each period, the high-quality firm optimally sets the highest possible price at which consumers purchase and rate with probability 1. Hence, $p_t^H = min\{q^H - e, E[q_t|R_{t-1}, p_t^H]\}$. We conclude that high-quality firms set this unique price, $p_t^H = min\{q^H - e, E[q_t|R_{t-1}, p_t^H]\}$, with probability 1.

We turn our attention to low-quality firms. First, we show that the price which the low-quality firm sets when it receive a good rating in any period, $t \in \{1, 2\}$, is unique. And this price is $p_{1,1}^L = p_{1,11}^L = q^L - e$. The argument here is the same as the argument used for when high-quality firms receive a good rating. The low-quality firm receives a good rating at any price weakly below $\overline{p_t^L} = q^L - e$, and $\overline{p_t^L} < q^L$. As consumers beliefs are weakly above q^L, they buy at any price weakly below $\overline{p_t^L}$. Since demand and ratings are the same for all prices weakly below $\overline{p_t^L}$, a low-quality firm obtaining a positive rating optimally sets $\overline{p_t^L}$ with probability 1. We can conclude that low-quality firms who obtain a rating in period t set $\overline{p_t^L}$ with probability 1.

Next, we consider the situation where the low-quality firm receives no rating for the first time. The related ratings history are $R_1 = \{0\}, R_2 = \{10\}$. We show that $p^L_{1,0} = E[q_1|R_0, p^H_1]$ and $p^L_{2,10} = E[q_2|R_1, p^H_2]$. We have shown above that all prices above $\overline{p^L_t}$ induces the belief $E[q_t|R_{t-1}, p^H_t]$ - as a result of Selection Assumption 2. Therefore, a low-quality firm receiving no rating optimally sets the highest possible price, $E[q_t|R_{t-1}, p^H_t]$ with probability 1.

Finally, consider the situation where the low-quality firm already has a history of receiving no rating. The related ratings history is $R_2 = \{00, 01\}$. Since by Selection Assumption 1 only low-quality firms receive no rating on the equilibrium path, consumer's belief on observing any history of no rating is that the firm is of a low-quality. Hence $p^L_{2,00} = p^L_{2,01} = E[q_2|R_1 = \{0\}] = q^L$.

This concludes the proof.

□

Lemma 11. *In the first period, consumer's beliefs for each equilibrium price p_1 is given by*

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta^*_1(1-\gamma)q^L}{\gamma + \delta^*_1(1-\gamma)} & \text{if } p_1 > \overline{p^L_1} \\ q^L & \text{if } p_1 \leq \overline{p^L_1} \end{cases},$$

in the second period,

$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta^*_1)\delta^*_2(1-\gamma)q^L}{\gamma + (1-\delta^*_1)\delta^*_2(1-\gamma)} & \text{if } p_2 > \overline{p^L_2} \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination} \end{cases},$$

and in the third period,

$$E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta^*_1)(1-\delta^*_2)(1-\gamma)q^L}{\gamma + (1-\delta^*_1)(1-\delta^*_2)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2 \end{cases}.$$

Proof of Lemma 11.
As with Lemma 3, we prove this lemma by constructing expected quality using Bayes rule. We start by considering the third period, followed by the second then the first.

We begin with the third period. In the third period, consumers are aware of historical ratings,

R_2 and current prices p_3. Given that this is the final period of the game, new ratings are not useful for firms - as there is no future period to signal to. By Selection Assumption 2, this implies that the expected quality in period 3 is independent of prices. Thus, the expected quality in period 3 is independent of prices and only depends on the rating history R_2. As a result, firms set the highest possible price and extract the full consumer surplus.

Note that the possible ratings history are $R_2 \in \{00, 01, 10, 11\}$. When consumers observe a ratings history of $\{00, 01, 10\}$, they expect that the firm is a low-quality firm (Selection Assumption 1).

Using Bayes rule, we pin down consumers' expectations on observing $R_2 = \{11\}$ in period 3. In equilibrium, low-quality firms receive no rating with some probability δ_t^* in period $t \in \{1, 2\}$. Hence, consumers observe a good rating with probability $\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)$ - and know that the probability of a high-quality firm is γ. Hence, $E[q_3|R_2 = 11] = \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$.

We conclude that

$$E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2 \end{cases}.$$

Next, we consider the second period. In the second period, consumers observe $R_1 \in \{0, 1\}$ and the price p_2. As with the third period, on observing $R_1 = 0$ Selection Assumption 1 implies that the firm is of a low-quality.

Using Bayes rule, we pin down consumers' expectations in equilibrium in period 2, conditional on $R_1 = 1$. We distinguish between two cases: the low-quality firm choosing between a good or no rating. When the low-quality firm receives no rating, it sets $p_2^L = E[q_2|R_1 = 1, p_2^H]$ and when it receives a good rating, $p_2^L = \overline{p_2^L} = q^L - e$. Note that $p_2^H = min\{q^H - e, E[q_2|R_1, p_2^H]\}$.

We now start with the first case, i.e. the low-quality firm sets $p_2^L = \overline{p_2^L} = q^L - e$ and gets a good rating. Since $q^H - e > q^L - e$ and $E[q_2|R_1 = 1, p_2^H] \geq q^L > q^L - e$, in the equilibrium, consumers who observe $q^L - e$ believe the firm is a low-quality firm, i.e. $E[q_2|R_1 = 1, p_2 = q^L - e] = q^L$.

Next, consider the second case where the low-quality firm sets, by Lemma 10, $p_2^L = E[q_2|R_1 = 1, p_2^H]$ and gets no rating. We distinguish two scenarios. First, suppose $p_2^H = min\{E[q_2|R_1 = 1, p_2^H], q^H - e\} = E[q_2|R_1 = 1, p_2^H]$. By Lemma 10, $p_{2,10}^L = E[q_2|R_1 = 1, p_2^H] = p_2^H$. On observing this price level, Bayes rule implies $E[q_2|R_1 = 1, p_2^H] = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$ in equilibrium. Second, consider the scenario where $p_2^H = min\{E[q_2|R_1 = 1, p_2^H], q^H - e\} = q^H - e$. Since $p_2^L = E[q_2|R_1, p_2^H] \neq q^H - e = p_2^H$, consumers who observe p_2^H believe $E[q_2|R_1, p_2^H] = q^H$. Because $p_2^L = E[q_2|R_1, p_2^H] = q_H > \overline{p_2^L}$ and $p_2^H = q^H - e > \overline{p_2^L}$, Selection Assumption 2 implies that both p_2^L and p_2^H induce the same equilibrium beliefs. But then we must have $\delta_2^* = 1$ in equilibrium. Note that beliefs $E[q_2|R_1, p_2^H] = q_H$ are then a special case of $E[q_2|R_1 = 1, p_2^H] = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$ for $\delta_2^* = 1$.

This concludes the second case.

We conclude that $E[q_2|R_1 = 1, p_2^H] = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$.

The final stage is to consider period 1. The proof for this is identical to proof in Lemma 11.

This concludes the proof.

$\square$

With Lemma 10 and 11, we can now prove Proposition 11.

Proof of Proposition 11.
We show that all equilibria satisfying our equilibrium Selection Assumptions exhibits similar characteristics as those in the main section of the paper. To do so, we show that a perfect Bayesian equilibrium exists, and that all equilibria satisfying our equilibrium Selection Assumptions satisfy similar properties as those in the main body. Specifically,

1. High-quality firms receive a good rating with probability 1 and charge $p_t^H = E[q_t|R_{t-1}, p_t^H]$, $t \in \{1, 2\}$.
2. Low-quality firms randomize their strategy in period $t \in \{1, 2\}$.
   a. They charge $\overline{p_t^L} = q^L - e$ and obtain a good rating with probability $1 - \delta_t^*$.
   b. They charge p_t^H and obtain no rating with probability δ_t^*.
3. Firms set prices equal to expected quality conditional on ratings in the last period.
4. Consumer beliefs given equilibrium prices are given by:
   a. In period 1, $E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} & \text{if } p_1 > \overline{p_1^L} \\ q^L & \text{if } p_1 \leq \overline{p_1^L} \end{cases}$

b. In period 2,

$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} & \text{if } p_2 > \overline{p_2^L} \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination} \end{cases}$$

c. In period 3, $E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2 \end{cases}$

Furthermore, we show that $\delta_1^* \in (0,1)$ and $\delta_2^* \in (0,1)$ is an equilibrium if $e < \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$, and $\delta_1^* \in (0,1)$ and $\delta_2^* = 1$ is an equilibrium if $e \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$. These equilibria exist if $e < \frac{(1-\gamma)(q^H - q^L)}{2-\delta_2^*}$, $\gamma > \frac{1}{3}$, and
$q^H - e \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}\}$.

From Lemmas 10 and 11, we have shown statement 4 and the low-quality firm's prices in statement 2. What remains, is to show statements 1 and 3, the mixed-strategy in statement 2, and to show the existence of the equilibrium.

We prove statement 3. In period 3, firms are no longer incentivized by future ratings. We have also shown in Lemma 10 that by Selection Assumption 2, in the final period consumers' beliefs are only dependent on past ratings and therefore independent of the price they observe in period 3. Firms set prices in period 3 equal to the expected quality conditional on past ratings. This concludes the proof of statement 3.

Next, we prove statement 1. From Lemma 10, we have shown that $p_t^H = min\{q^H - e, E[q_t|R_{t-1}, p_t^H]\}$. To show that $min\{q^H - e, E[q_t|R_{t-1}, p_t^H]\} = E[q_t|R_{t-1}, p_t^H]$, suppose towards a contradiction that $min\{q^H - e, E[q_t|R_{t-1}, p_t^H]\} = q^H - e$. We do this in two parts, for period 2 then period 1.

Note first that in period 2, the low-quality firm with no rating in period 1 always sets the price q^L in periods 2 and 3. Thus, we only consider histories after which the low-quality firm received a good rating in period 1. We know from Lemma 10 that $p_{2,10}^L = E[q_2|R_1, p_2^H]$. Thus, in the candidate equilibrium we have $p_2^H \neq p_{2,10}^L$ and only high-quality firms set $p_2^H = q^H - e$, which is why consumers believe that $E[q_2|R_1 = 1, p_2^H] = q^H$. Given Selection Assumption 2, for any price above $q^L - e$ consumers beliefs are the same. Hence $p_{2,10}^L = E[q_2|R_1 = 1, p_2^H] = q^H$. These beliefs are only correct in equilibrium if $\delta_2^* = 0$. But we cannot have $\delta_2^* = 1$ in equilibrium. To see why, note that in period 2 in the candidate equilibrium the low-quality firms charges a low price and receives a good rating, earning in periods 2 and 3 together $q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$. Deviating by setting a high price q_H gives no rating, but earns in periods 2 and 3 the profit $q^H + q^L$. Since the firm earns the same profits in either case in period 1, the deviation is profitable if it increases profits in periods 2 and 3. Because $q^L > q^L - e$ and $q^H > \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$, this deviation is profitable, contradicting $\delta_2^* = 0$ and that $p_2^H = q^H - e$.

We conclude that $p_2^H = E[q_2|R_1, p_2^H]$.

In period 1, we know from Lemma 10 that $p_{1,0}^L = E[q_1|R_0, p_1^H]$. In the candidate equilibrium, we have $p_1^H \neq p_{1,0}^L$ and only high-quality firms charge $p_1^H = q^H - e$, which is why consumers believe that $E[q_1|R_0, p_1^H] = q^H$. Given Selection Assumption 2, for any price above $q^L - e$ consumers beliefs are the same. Hence $p_{1,0}^L = E[q_1|R_0, p_1^H] = q^H$. These beliefs are only correct in equilibrium if $\delta_1^* = 0$. We now show that we cannot have $\delta_1^* = 0$ in equilibrium. To see why, note that in period 1 in the candidate equilibrium, the low-quality firm sets a low price, receives a good rating and earns total expected profits $q^L - e + (1-\delta_2^*)(q^L - e + E[q_3|R_2 = 11]) + \delta_2^*(E[q_2|R_1 = 1, p_2^H] + q^L)$. If the firm deviates in period 1 to price q^H and charges q^L in subsequent periods, it will earn $q^H + q^L + q^L$. Because $q^L > q^L - e$, $q^L > (1-\delta_2^*)(q^L - e) + \delta_2^*(q^L)$, and $q^H > (1-\delta_2^*)(E[q_3|R_2 = 11]) + \delta_2^*(E[q_2|R_1 = 1, p_2^H])$, this deviation is profitable, contradicting $\delta_1^* = 0$ and that $p_1^H = q^H - e$.

We conclude that $p_1^H = E[q_1|R_0, p_1^H]$.

Overall, this prove statement 1, i.e. that $p_t^H = min\{q^H - e, E[q_t|R_{t-1}, p_t^H]\} = E[q_t|R_{t-1}, p_t^H]$.

We have now shown that statements 1,3 and 4 hold in equilibria that satisfy our Selection Assumptions. We continue to show statement 2 by characterizing the (mixed) strategies of low-quality firms.

We now characterize the low-quality firms' mixed-strategy in equilibrium in period 2, i.e. δ_2^*. Recall that δ_2^* is the probability of setting a high price of p_2^H that leads to a no rating. Reversely, $1 - \delta_2^*$ is the probability of setting a low price of $q^L - e$ that leads to a good rating. Note that low-quality firms only mix prices after histories where they received a good rating in period 1. This is because consumers' beliefs are such that on observing a period of no rating, they believe the firm to be a low-quality firm.

After a history of a good rating in period 1, when the low-quality firm sets a price of $q^L - e$ in period 2, it obtains a good rating and earns in periods 2 and 3

$$q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}. \tag{16}$$

This is strictly increasing in δ_2^*.

Alternatively, if the low-quality firm sets the high price p_2^H it obtains no rating in period 2 and earns in periods 2 and 3

$$\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L. \tag{17}$$

This is strictly decreasing in δ_2^*.

We now show that $\delta_2^* = 1$ is only possible if no mixed-strategy exists in period 2. First, consider that $\delta_2^* = 1$. Then (16) and (17) become $q^L - e + q^H$ and $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)} + q^L$ respectively. In order for $\delta_2^* = 1$ to be optimal, we must have that at $\delta_2^* = 1$, (17) is weakly larger than (16), i.e.

$$\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)} + q^L \geq q^L - e + q^H \Leftrightarrow e \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)} \tag{18}$$

Observe that (16) is strictly increasing in δ_2^* and (17) is strictly decreasing in δ_2^*, which is why (18) implies that (17) is larger than (16) for all $\delta_2^* \leq 1$. Therefore, if (18) is met, $\delta_2^* = 1$ is an equilibrium and no mixed-strategy equilibrium exists in period 2.

We now show that $\delta_2^* = 0$ is not a possible equilibrium in period 2. Suppose towards a contradiction that $\delta_2^* = 0$. Then (16) and (17) become $q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ and $q^H + q^L$ respectively. Since $q^H > \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ and $q^L > q^L - e$, the firm is strictly better of by deviating, setting a high price and receiving no rating, which contradicts $\delta_2^* = 0$. We conclude that $\delta_2^* = 0$ cannot be an equilibrium.

We now show that for

$$e < \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}, \tag{19}$$

there is a unique $\delta_2^* \in (0,1)$ that characterizes the low-quality firms mixed-strategy in period 2.

Recall that a mixed-strategy equilibrium only exists when (16) and (17) are equal. Observe that for $\delta_2^* = 1$, (17) is strictly below (16) if and only if (19) holds. Next, observe that for $\delta_2^* = 0$, (17) is strictly above (16) because $q^L + q^H > q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$.

Since (16) is strictly increasing and (17) is strictly decreasing in δ_2^*, there is a unique $\delta_2^* \in (0,1)$ such that (16) equals (17) if and only if (19). Otherwise, i.e. if and only if (18), we have $\delta_2^* = 1$.

We now pin down δ_2^*. Note that if $\delta_2^* < 1$, it is determined by (16) equal (17), i.e.

$$q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L$$

We have two candidates that solve this equation:

$$\begin{aligned}\delta_2^* =& \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)e} \pm \\ & \frac{[(2\gamma(q^H - q^L))^2 + ((1-\delta_1^*)(1-\gamma)+2\gamma)^2 e^2]^{\frac{1}{2}}}{2(1-\delta_1^*)(1-\gamma)e}\end{aligned}$$

Recall that $\delta_2^* \in (0,1)$. In the subtraction case,

$$\begin{aligned}& \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)e} - \frac{[(2\gamma(q^H - q^L))^2 + ((1-\delta_1^*)(1-\gamma)+2\gamma)^2 e^2]^{\frac{1}{2}}}{2(1-\delta_1^*)(1-\gamma)e} \\ & < \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)e} - \frac{1}{2} \\ & = -\frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)e} < 0\end{aligned}$$

Therefore, we conclude that

$$\begin{aligned}\delta_2^* =& \frac{1}{2} - \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)e} + \\ & \frac{[(2\gamma(q^H - q^L))^2 + ((1-\delta_1^*)(1-\gamma)+2\gamma)^2 e^2]^{\frac{1}{2}}}{2(1-\delta_1^*)(1-\gamma)e}\end{aligned}$$

Further, $\delta_2^* > \frac{1}{2} + \frac{\gamma(q^H - q^L)}{(1-\delta_1^*)(1-\gamma)e} - \frac{2\gamma(q^H - q^L)}{2(1-\delta_1^*)(1-\gamma)e} = \frac{1}{2}$.

Therefore, $\delta_2^* \in (\frac{1}{2}, 1)$ if and only if $e < \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$, and $\delta_2^* = 1$ otherwise.

We now turn our attention to period 1 and characterize δ_1^*. We first show that δ_1^* is unique. Recall that in period 1, the low-quality firm charges a low price $q^L - e$ with probability $1 - \delta_1^*$ and obtains a rating, and it charges a high price $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$ with probability δ_1^* and obtains no rating in period 1.

When the low-quality firm charges $q^L - e$, the total continuation profit of the low-quality firm is

$$\pi_1^L(R_1=1) = q^L - e + (1-\delta_2^*)[q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}] + \delta_2^*[\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L], \tag{20}$$

where the first term are the profits in period 1, the second term are the continuation profits if the firm charges a low price and obtains a good rating in period 2, and the third term are the continuation profits if the firm charges a high price and obtains no rating in period 2.

We show that (20) is strictly increasing in δ_1^* if $\gamma > \frac{1}{3}$.

To show that (20) is strictly increasing in δ_1^*, we first show that $\frac{\partial \delta_2^*}{\partial \delta_1^*} > 0$.

Taking the derivative of (16) and (17) and solving for $\frac{\partial \delta_2^*}{\partial \delta_1^*}$. We find that

$$\frac{\partial \delta_2^*}{\partial \delta_1^*} = \frac{\delta_2^*(\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 - (1-\delta_2^*)(\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2}{(1-\delta_1^*)((\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2 + (\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2)}. \tag{21}$$

Notice that the denominator is positive. The numerator simplifies to $(2\delta_2^* - 1)(\gamma^2 - (1-\delta_1)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2)$. We show that if $\gamma > \frac{1}{3}$, we have $(2\delta_2^* - 1)(\gamma^2 - (1-\delta_1)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2) > 0$. Since $\delta_2^* \in (\frac{1}{2}, 1]$, $2\delta_2^* - 1 > 0$. We know that $\delta_1^* \in (0, 1]$ and that $\delta_2^*(1-\delta_2^*)$ is maximum at $\delta_2^* = 0.5$. Therefore, $\gamma^2 - (1-\delta_1)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2 > \gamma^2 - 0.25(1-\gamma)^2$ and $\gamma^2 - 0.25(1-\gamma)^2 > 0$ whenever $\gamma > \frac{1}{3}$. Thus, if $\gamma > \frac{1}{3}$, we have $(2\delta_2^* - 1)(\gamma^2 - (1-\delta_1)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2) > 0$ and (21) > 0. $\gamma > \frac{1}{3}$ is a sufficient condition.

Next, we show that total derivative of (20) with respect to δ_1^* is greater than 0.

$$\frac{\partial \pi_1^L(R_1=1)}{\partial \delta_1^*} = \frac{\partial \delta_2^*}{\partial \delta_1^*}[e - \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} + \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}] + \frac{\gamma(1-\gamma)\delta_2^*(q^H - q^L)(\delta_2^* - \frac{\partial \delta_2^*}{\partial \delta_1^*}(1-\delta_1^*))}{(\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma))^2} + \frac{\gamma(1-\gamma)(1-\delta_2^*)(q^H - q^L)(\frac{\partial \delta_2^*}{\partial \delta_1^*}(1-\delta_1^*) + (1-\delta_2^*))}{(\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2}$$

We show that $\frac{\partial \pi_2^L}{\partial \delta_1^*} > 0$ in three parts.

Consider first that $\frac{\partial \delta_2^*}{\partial \delta_1^*}e$ and show that this is greater than 0. Since $\frac{\partial \delta_2^*}{\partial \delta_1^*} > 0$ when $\gamma > \frac{1}{3}$, we can conclude that $\frac{\partial \delta_2^*}{\partial \delta_1^*}e > 0$.

Second, we reformulate $\frac{\partial \pi_2^L}{\partial \delta_1^*}$.

To do so, consider $\frac{\gamma(1-\gamma)(1-\delta_2^*)(q^H-q^L)(\frac{\partial\delta_2^*}{\partial\delta_1^*}(1-\delta_1^*)+(1-\delta_2^*))}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2}+\frac{\gamma(1-\gamma)\delta_2^*(q^H-q^L)(\delta_2^*-\frac{\partial\delta_2^*}{\partial\delta_1^*}(1-\delta_1^*))}{(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}$. Substituting (21), simplifies to $\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}$. This leaves us with the simplified equation of

$$\begin{aligned}\frac{\partial\pi_1^L(R_1=1)}{\partial\delta_1^*}=&\frac{\partial\delta_2^*}{\partial\delta_1^*}[e-\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}+\\&\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}]+\\&\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}.\end{aligned}$$

Finally, since $\frac{\partial\delta_2^*}{\partial\delta_1^*}e>0$, we show that $\frac{\partial\delta_2^*}{\partial\delta_1^*}[\frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}-\frac{\gamma q^H+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}]+\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}>0$. Substituting (21), we get

$$\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}[1+\\\frac{(1-\delta_1^*)(1-2\delta_2^*)(\delta_2^*(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2-(1-\delta_2^*)(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))}]$$

We check that this is positive. Notice that $\frac{\gamma(1-\gamma)(q^H-q^L)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2+(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2}>0$. Therefore, what remains is to show $1+\frac{(1-\delta_1^*)(1-2\delta_2^*)(\delta_2^*(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))^2-(1-\delta_2^*)(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))^2)}{(\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma))(\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma))}>0$, and this is true if and only if

$$\begin{aligned}&(1-\delta_1^*)(1-2\delta_2^*)^2[\gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2]\\<&\gamma^2+\gamma(1-\gamma)(1-\delta_1^*)+(1-\gamma)^2(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*).\end{aligned}$$

We can verify that this is always true. Since $\delta_1^*\in[0,1]$ and $\delta_2^*\in[\frac{1}{2},1]$, $\delta_1^*(1-2\delta_2^*)^2\in[0,1]$. Therefore, $(1-\delta_1^*)(1-2\delta_2^*)^2[\gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2]<\gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2$. It is easy to see that $\gamma^2-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2<\gamma^2+\gamma(1-\gamma)(1-\delta_1^*)+(1-\gamma)^2(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)$. For $-(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)(1-\gamma)^2<\gamma(1-\gamma)(1-\delta_1^*)+(1-\gamma)^2(1-\delta_1^*)^2\delta_2^*(1-\delta_2^*)$, the left hand side is negative and the right hand side is positive.

We conclude that $\frac{\partial\pi_1^L(R_1=1)}{\partial\delta_1^*}>0$ when $\gamma>\frac{1}{3}$. Note that $\gamma>\frac{1}{3}$ is a sufficient but not necessary condition.

Next consider the situation when the low-quality firm charges a high price $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ with probability δ_1^* and obtains no rating in period 1. The firm then charges q^L in subsequent periods. Thus, the total continuation profits are given by

$$\pi_1^L(R_1 = 0) = \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} + q^L + q^L \tag{22}$$

This is strictly decreasing in δ_1^*.

We now show that $\delta_1^* = 1$ is only possible if no mixed-strategy equilibrium in period 1 exists. Suppose that $\delta_1^* = 1$. Then (20) and (22) become $q^L - e + (1-\delta_2^*)[q^L - e + q^H] + \delta_2^*[q^H + q^L]$ and $\gamma q^H + (1-\gamma)q^L + q^L + q^L$ respectively. For $\delta_1^* = 1$ to be optimal, we must have (20) lower that (22) for $\delta_1^* = 1$,

$$\begin{aligned} q^L - e + (1-\delta_2^*)[q^L - e + \delta_2^*[q^L] + q^H &\leq \gamma q^H + (1-\gamma)q^L + q^L + q^L \\ \iff (2-\delta_2^*)e &\geq (1-\gamma)(q^H - q^L). \end{aligned}$$

Further, when $\gamma > \frac{1}{3}$, (20) increases in δ_1^* and (22) decreases in δ_1^*. Hence, when $(2-\delta_2^*)e > (1-\gamma)(q^H - q^L)$, (22) is larger than (20) for all $\delta_1^* \leq 1$. This indicates that $\delta_1^* = 1$ is an equilibrium and no mixed-strategy equilibrium exists in period 1. We conclude that $\delta_1^* = 1$ can only be an equilibrium if no mixed-strategy equilibrium exists.

We now show that $\delta_1^* = 0$ is not an equilibrium. Suppose towards a contradiction that $\delta_1^* = 0$. Then (20) and (22) become $q^L - e + (1-\delta_2^*)[q^L - e + \frac{\gamma q^H + (1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_2^*)(1-\gamma)}] + \delta_2^*[\frac{\gamma q^H + \delta_2^*(1-\gamma)q^L}{\gamma + \delta_2^*(1-\gamma)} + q^L]$ and $q^H + q^L + q^L$ respectively. Since $q^L - e < q^L$, $\frac{\gamma q^H + (1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_2^*)(1-\gamma)} < q^H$, and $\frac{\gamma q^H + \delta_2^*(1-\gamma)q^L}{\gamma + \delta_2^*(1-\gamma)} < q^H$ then (20) $<$ (22). Therefore, if $\delta_1^* = 0$ and the low-quality firm participates in ratings harvesting with probability 1, deviating to the high price and obtaining no ratings is profitable. This contradicts $\delta_1^* = 0$. We conclude that $\delta_1^* = 0$ cannot be an equilibrium.

We now show that for

$$(2-\delta_2^*)e < (1-\gamma)(q^H - q^L), \tag{23}$$

there is a unique $\delta_1^* \in (0,1)$ that characterises the low-quality firms mixed-strategy in period 1. Recall that a mixed-strategy equilibrium only exists when (20) and (22) are equal. First, observe that for $\delta_1^* = 1$, (22) is strictly below (20). Next, observe that $\delta_1^* = 0$, (22) is strictly above (20). Since (20) is strictly increasing in δ_1^* when $\gamma > \frac{1}{3}$ and (22) is strictly decreasing in δ_1^*, there is a unique $\delta_1^* \in (0,1)$ such that (20) and (22) are equal if (23) and $\gamma > \frac{1}{3}$ hold.

We have shown that we cannot have an equilibrium where $\delta_1^* = 0$ or $\delta_2^* = 0$, and that $\delta_2^* = 1$ can be an equilibrium if no mixed-strategy equilibrium exists. Additionally, if $\gamma > \frac{1}{3}$, then $\delta_1^* = 1$ can be an equilibrium if no mixed-strategy equilibrium exists and if a mixed-strategy exists, $\delta_1^* \in (0,1)$ exists such that (20) and (22) are equal, it is unique if $\gamma > \frac{1}{3}$. And, if $\delta_2^* \in (0,1)$ exists such that (16) and (17) are equal, it must be unique. Therefore, we either have a unique pure-strategy in equilibrium or a unique mixed-strategy in equilibrium in each period when $\gamma > \frac{1}{3}$. We have also characterised the mixed-strategy equilibrium for period 2 and shown that $\delta_2^* \in (\frac{1}{2}, 1]$. We conclude that statement 2 holds, and therefore conclude that statements 1-4 hold in equilibrium.

We now show that equilibria satisfying statements 1-4 indeed exist.

We begin by considering the scenario where (19), (23), $q^H - e \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}\}$ and $\gamma > \frac{1}{3}$ hold together.

Consumers' beliefs are as follows. In period 1,

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} & \text{if } p_1 > q^L - e \\ q^L & \text{if } p_1 \leq q^L - e, \end{cases}$$

and in period 2,

$$E[q_2|R_1, p_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} & \text{if } p_2 > q^L - e \text{ and } R_1 = 1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination,} \end{cases}$$

and in period 3,

$$E[q_3|R_2] = \begin{cases} \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)} & \text{if } R_2 = \{11\} \\ q^L & \text{for any other } R_2. \end{cases}$$

These beliefs follow Bayes rule on the candidate equilibrium path of play. The candidate equilibrium is consistent with our Selection Assumptions. Because high-quality firms obtain a good rating with probability 1, the candidate equilibrium is consistent with Selection Assumption 1. Further, whenever low-quality firms obtain no rating, consumers' beliefs are independent of prices (in both periods 1 and 2), and in period 3, beliefs are the same for all period 3 prices and they only depend on the history of ratings. Therefore, the candidate equilibrium is consistent with Selection Assumption 2.

The candidate equilibrium is such that high-quality firms always play $p_t^H = E[q_t|R_{t-1}, p_t^H]$ for all t and get a good rating. Low-quality firms mix between playing a low price $q_L - e$ and getting a good rating with probability δ_t^*, and setting a high price p_t^H and getting no rating with probability $(1 - \delta_t^*)$ in each period $t \in \{1, 2\}$.

More precisely, in the candidate equilibrium in period 1, the high-quality firm sets a price $p^H_{1,1} = \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and obtains a good rating with probability 1. In period 2, conditional on receiving a good rating in period 1, the high-quality firm sets a price $p^H_{2,11} = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$ and obtains a good rating with probability 1. If the high-quality firm did not receive a good rating in period 1, consumers believe the firm is of a low quality and thus the maximum price that the high-quality firm can set is q^L and receives a good rating. To see that the firm receives a good rating, note that by assumption $q^H - e \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}\}$ and both of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and $\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$ are larger than q^L. Therefore, when consumers pay q^L for a high quality product, they receive a sufficient amount of excess surplus and leave a good rating. In the third period of the candidate equilibrium, having received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, the high-quality firm sets a price of $\frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$. If the firm did not receive a continuation of good ratings, i.e. a rating history of $R_2 \in \{00, 01, 10\}$, then it sets a maximum price of q^L.

In period 1 of the candidate equilibrium, the low-quality firm sets $p^L_{1,0} = \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and obtains no rating with some probability δ_1^*, and $p^L_{1,1} = q^L - e$ and obtains a good rating with some probability $1-\delta_1^*$. In period 2, conditional on having obtained a good rating in period 1, the low-quality firm sets a price $p^L_{2,11} = q^L - e$ with some probability $1 - \delta_2^*$ and obtains a good rating, and a price $p^L_{2,10} = \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}$ with some probability δ_2^* and obtains no rating. If the low-quality firm obtained no rating in period 1, then it sets a price q^L in period 2 and obtains no rating. In period 3, if the low-quality firm received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, it sets a price of $\frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$, and when it receives a rating history of $R_2 = \{00, 10, 01\}$, it sets a price of q^L. This fully characterizes the firms' prices and consumers' beliefs.

We now show that the firms have no profitable deviations.

In the candidate equilibrium, the high-quality firm earns a total profit of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$.

In period 3, deviations to a higher price would reduce demand to zero and earn a total maximum profit of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)} + 0$, and deviations to a lower price would reduce profit margins in the third period without increasing demand. Neither deviation increases profits.

In period 2, deviations to a higher price would reduce demand to zero and result in no rating, this leads to a total maximum profit of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + 0 + q^L$, and deviations to a lower price would reduce profit margins in the second period without increasing demand.

Neither deviation increases profits.

In period 1, deviations to a higher price would reduce demand to zero and result in no rating, this leads to a total maximum profit of $0 + q^L + q^L$, and deviations to a lower price would reduce profit margins in the first period without increasing demand. Neither deviation increases profits.

Therefore, there are no profitable deviations for the high-quality firm in any period.

Next, we show that low-quality firms have no profitable deviation.

In the candidate equilibrium, the low-quality firm earns a total profit of $q^L - e + q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$. For the equilibrium $\delta_1^* \in (0, 1)$ and $\delta_2^* \in (0, 1)$, the low-quality firm is indifferent between setting the lower price that obtains good rating and setting a higher price that obtains no rating in all periods. Hence, the firm is indifferent between the total profits of $q^L - e + q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$, $q^L - e + \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} + q^L + q^L$. Recall that (19) implies $\delta_2^* \in (0, 1)$ and (23) implies $\delta_1^* \in (0, 1)$, and $\gamma > \frac{1}{3}$ implies $\delta_1^* \in (0, 1)$ is unique. Showing that there exists no profitable deviation from any of these cases shows that there is no profitable deviation for the low-quality firm.

In period 3, deviations towards a price above $\frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$ would result in zero demand and a maximum total profit of $q^L - e + q^L - e + 0$, and a deviation towards a lower price results in smaller profit margins without increasing demand. Hence, there is no profitable deviation in period 3.

In period 2, the low-quality firm is indifferent between setting the price $q^L - e$ and $\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$, where $\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} > q^L - e$ and her total profits are $q^L - e + q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$ and $q^L - e + \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)} + q^L$ respectively. Deviations towards a price above $\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$ leads to zero demand in period 2 and the maximal profits of $q^L - e + 0 + q^L$, which is not a profitable deviation. Deviations towards a price $p_2 \in (q^L - e, \frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)})$, leads to the same rating as when the firm sets $\frac{\gamma q^H + (1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma + (1-\delta_1^*)\delta_2^*(1-\gamma)}$; hence, any deviation to p_2 lowers margin without improving demand or future profit, and are therefore not profitable. Deviations towards a price below $q^L - e$ decreases profits in period 2 but does not increase demand or change the rating the firm receives when setting a price of $q^L - e$, which is why this is also not a profitable deviation. Therefore, in period 2, there is no profitable deviation for the firm.

In period 1, the low-quality firm is indifferent between setting the price $q^L - e$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)}$, where its total profits are $q^L - e + q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)q^L}{\gamma + (1-\delta_1^*)(1-\delta_2^*)(1-\gamma)}$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma + \delta_1^*(1-\gamma)} + q^L + q^L$

respectively. If it deviates to a price above $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$, demand falls to zero and it makes a total profit of $0+q^L+q^L$, which is not a profitable deviation. If it deviates to a price $p_1 \in (q^L-e, \frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)})$, then it receives the same rating as when it sets the price of $\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$; hence such a deviation reduces margins in the first period without improving demand or future profit and is not profitable. When deviating to a price below q^L-e, the firm receives a good rating; however, the deviation does not increase demand and her margins are lower than when setting the price of q^L-e, which is why the deviation is not profitable.

We conclude that there are no profitable deviations for either high- or low-quality firms from the candidate equilibrium.

We conclude that if $e < \frac{(1-\delta_1^*)(1-\gamma)(q^H-q^L)}{\gamma+(1-\delta_1^*)(1-\gamma)}$, $e < \frac{(1-\gamma)(q^H-q^L)}{2-\delta_2^*}$, $\gamma > \frac{1}{3}$ and $q^H-e \geq max\{\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H+(1-\delta_1^*}{\gamma+(1-\delta_1^*)}$ the candidate equilibrium exists.

Consider next the case where (18), (23), $\gamma > \frac{1}{3}$ and $q^H-e \geq max\{\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H+(1-\delta_1^*)\delta_2^*(1-\gamma)q^L}{\gamma+(1-\delta_1^*)\delta_2^*(1-\gamma)}\}$. In this scenario, $\delta_2^*=1$. The last three inequalities are identical to the previous case and play the same role in this case.

In the candidate equilibrium, consumers' beliefs are as follows. In period 1,

$$E[q_1|p_1] = \begin{cases} \frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} & \text{if } p_1 > q^L-e \\ q^L & \text{if } p_1 \leq q^L-e, \end{cases}$$

and in period 2,

$$E[q_2|R_1,p_2] = \begin{cases} \frac{\gamma q^H+(1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)} & \text{if } p_2 > q^L-e \text{ and } R_1=1 \\ q^L & \text{for any other } p_2,\ R_1 \text{ combination,} \end{cases}$$

and in period 3,

$$E[q_3|R_2] = \begin{cases} q^H & \text{if } R_2=\{11\} \\ q^L & \text{for any other } R_2. \end{cases}$$

These beliefs follow Bayes rule on the candidate equilibrium path of play. The candidate equilibrium is consistent with our Selection Assumptions. Because high-quality firms obtain a good rating with probability 1, the candidate equilibrium is consistent with Selection Assumption 1. Further, whenever low-quality firms obtain no rating, consumers' beliefs are independent of prices (in both periods 1 and 2), and in period 3, beliefs are the same for all prices. Therefore, the candidate equilibrium is consistent with Selection Assumption 2.

In the candidate equilibrium in period 1, the high-quality firm sets a price $p^H_{1,1} = \frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and obtains a good rating with probability 1. In period 2, conditional on receiving a good rating in period 1, the high-quality firm sets a price $p^H_{2,11} = \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)}$ and obtains a good rating with probability 1. If the high-quality firm did not receive a rating in period 1, consumers believe the firm is of a low quality and thus the maximum price that the high-quality firm can set is q^L and the firm receives a good rating. To see that the firm receives a good rating, note that by assumption $q^H - e \geq max\{\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)}\}$ and both of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)}$ are larger than q^L. In the third period of the candidate equilibrium, having received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, the high-quality firm sets a price of q^H. If the firm did not receive a continuation of good ratings, i.e. a rating history of $R_2 \in \{00, 01, 10\}$, then it sets a maximum price of q^L.

In period 1 of the candidate equilibrium, the low-quality firm sets a price $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}$ and obtains no rating with some probability δ_1^* and $q^L - e$ and obtains a good rating with some probability $1 - \delta_1^*$. In period 2, the low-quality firm sets a price of $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)}$ with probability 1 and receives no rating. In period 3, if the low-quality firm received a continuation of good ratings, i.e. a rating history of $R_2 = 11$, it sets a price of q^H, and when it receives a rating history of $R_2 = \{00, 10, 01\}$, it sets a price of q^L.

We now show that the firms have no profitable deviations.

In the candidate equilibrium, the high-quality firm earns a total profit of $\frac{\gamma q^H + \delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)} + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)} + q^H$. In period 3, deviations to a higher price reduces demand to zero, and deviations to a lower price reduces profit margins without increasing demand, thus there is no profitable deviation in period 3. In period 2, deviations to a higher price reduces demand to zero, which induces no rating and also reduces continuation profits; deviations to a lower price reduces profit margins without increasing demand. Thus, neither deviation is profitable in period 2. In period 1, deviations to a higher price reduces demand to zero, induce no rating and therefore reduces continuation profits in all future periods to q^L, and deviations to a lower price reduces profit margins without increasing demand. Neither deviation is profitable in period 1. Therefore, there are no profitable deviations for the high-quality firm.

In the candidate equilibrium, the low-quality firm earns a total profit of $q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)} + q^L$. For the equilibrium $\delta_1^* \in (0, 1)$, the low-quality firm is indifferent between setting a lower price that obtains a good rating and setting a higher price that obtains no rating in period 1, and for $\delta_2^* = 1$, the low-quality firm always sets a high price and obtains no rating in period 2. The low-quality firm is therefore indifferent between the total profits of $q^L - e + \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma) q^L}{\gamma + (1-\delta_1^*)(1-\gamma)} + q^L$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma) q^L}{\gamma + \delta_1^*(1-\gamma)} + q^L + q^L$. Recall that (18) implies $\delta_2^* = 1$, (23) implies $\delta_1^* \in (0,1)$, and $\gamma > \frac{1}{3}$ implies $\delta_1^* \in (0,1)$ is unique. In period 3, deviations towards a higher price, above q^L, reduces demand to zero, and deviations towards a lower price results in a lower profit margin without improving demand, which is why there is no profitable deviation in period 3.

In period 2, conditional on receiving a good rating in period 1, deviations towards a price above $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma) q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$ leads to zero demand and results in the same rating (and hence future profit) as setting a price of $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma) q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$, which is why it is not a profitable deviations. Further, deviations towards a price of $p_2 \in (q^L - e, \frac{\gamma q^H + (1-\delta_1^*)(1-\gamma) q^L}{\gamma + (1-\delta_1^*)(1-\gamma)})$ results in the same rating (and hence future profit) as setting a price of $\frac{\gamma q^H + (1-\delta_1^*)(1-\gamma) q^L}{\gamma + (1-\delta_1^*)(1-\gamma)}$, therefore this reduces margins without providing any additional continuation profit, and is not a profitable deviation. Deviations towards a price $p_2 \leq q^L - e$ leads to a maximal total profit of $q^L - e + q^L - e + q^H$, but since (18) holds, this is not a profitable deviation. Therefore, there are no profitable deviations in period 2 conditional on receiving a good rating in period 1. In period 2, conditional on having received no rating in period 1, deviations towards a price above q^L leads to zero demand and does not change rating and continuation profits, which is why this is not a profitable deviation. Further, since consumers beliefs on observing a single period of no rating is that the firm is of a low-quality, setting a price lower than q^L only reduce the margins in period 2 without increasing demand or future profits, thus this is not a profitable deviation. Therefore, there are no profitable deviations in period 2 conditional on not having received a rating in period 1.

In period 1, the low-quality firm is indifferent between setting the price $q^L - e$ and $\frac{\gamma q^H + \delta_1^*(1-\gamma) q^L}{\gamma + \delta_1^*(1-\gamma)}$. If it deviates to a price above $\frac{\gamma q^H + \delta_1^*(1-\gamma) q^L}{\gamma + \delta_1^*(1-\gamma)}$, demand falls to zero, it gets no rating and it makes a total profit of $0 + q^L + q^L$, which is not a profitable deviation. If it deviates to a price $p_1 \in (q^L - e, \frac{\gamma q^H + \delta_1^*(1-\gamma) q^L}{\gamma + \delta_1^*(1-\gamma)})$, then it receives the same rating as when it sets the price of $\frac{\gamma q^H + \delta_1^*(1-\gamma) q^L}{\gamma + \delta_1^*(1-\gamma)}$; hence such a deviation reduces margins in the first period without improving demand or future profit and is not a profitable deviation. When deviating to a price below $q^L - e$, the firm receives a good rating; however, the deviation does not increase demand and her margins are lower than when setting the price of $q^L - e$. Therefore, we conclude that there is no profitable deviation in period 1.

We conclude that there are no profitable deviations for either the high- or low- quality firm from the candidate equilibrium.

We conclude that if $e \geq \frac{(1-\delta_1^*)(1-\gamma)(q^H - q^L)}{\gamma + (1-\delta_1^*)(1-\gamma)}$, $e < (1-\gamma)(q^H - q^L)$, $\gamma > \frac{1}{3}$ and $q^H - e \geq$

$max\{\frac{\gamma q^H+\delta_1^*(1-\gamma)q^L}{\gamma+\delta_1^*(1-\gamma)}, \frac{\gamma q^H+(1-\delta_1^*)(1-\gamma)q^L}{\gamma+(1-\delta_1^*)(1-\gamma)}\}$, the candidate equilibrium exists.

This concludes the proof. □

B.3 Deriving Rating Utility

We derive the rating utility function from the utility function proposed in Rabin (1993). In his paper, Rabin proposes a utility function which incorporates a reciprocity term in addition to a consumption utility. This reciprocity term depends on the additional surplus that some player i is allowed to obtain given the actions of another player j relative to a predefined equity point.

$$U_i = \pi_i + [\frac{\pi_i - \pi_i^e}{\pi_i^H - \pi_i^{min}}][1 + \frac{\pi_j - \pi_j^e}{\pi_j^H - \pi_j^{min}}]$$

- For all $h \in \{i, j\}$.
- π_h is the utility.
- π_h^{min} is the lowest possible payoff to player h.
- π_h^H is the highest possible pareto efficient payoff to player h.
- $\pi_h^e = \frac{\pi_h^H + \pi_h^L}{2}$, where π_h^L denotes the lowest possible pareto efficient payoff to player h. π_h^e is the equitable reference point.

At this junction, allow us to provide some intuition. Suppose that player i is the consumer. Then this function takes into account the consumption utility and some additional reciprocity term. The additional term is what we consider the rating utility. Suppose a firm sets a low price, such that $\pi_i - \pi_i^e > 0$, the consumer would believe that the firm is treating him kindly. In response, consumers will receive a higher overall utility if he is kind to the firm, $\pi_j - \pi_j^e > 0$. In our context, a good rating will result in π_j being higher and therefore by leaving a good rating, consumers would be being kind to the firms.

On the contrary, a firm charging a high price such that pay off for the consumer is below the equitable point would result in consumers punishing the firm by lowering their profits in the future periods - perhaps through a negative rating. For simplicity, we remove the ability of consumers to punish a firm and assume that consumers are only able to provide good or no ratings. Specifically, in our model, we show that a good rating results in a better future pay off and hence satisfies this feature.

In what follows, I show how we adapt this framework to better fit our context. Firstly, we have assumed that providing a rating is costly for consumers. This seems to be an intuitive feature of our model and follows from the literature of costly provision of reviews and ratings (Avery et al., 1999; Miller et al., 2005). Secondly, we make some simplifications that make the framework more tractable in our setup. We remove the normalization terms in the denominator and remove the "1".[40] Thirdly, we consider that the rating component of the utility only comes into effect when a good rating is provided. This leaves us with the following function:

$$U_i = \pi_i + \mathbb{1}_{\{R_t=1\}}\{[\pi_i - \pi_i^e][\pi_j - \pi_j^e] - e\}$$

Since $\pi_i = q^j - p_1^j$, $\pi_i^e = \frac{(q^j - q^j)+(q^j - 0)}{2}$. The highest possible pareto payoff to consumers being $q^j - 0$ where sellers set a price of 0 and the lowest possible being 0, where sellers set a price of q^j.

Moreover, $\pi_j = p_1 + p_2$, profits of the firm being the sum of profits in two periods, given 0 marginal cost, profits is the sum of prices in both periods. And $\pi_j^e = \frac{(p_1+q^H)+(p_1+q^L)}{2}$. The seller, setting some price p_1 in period 1, is able to get a maximum benefit of q^H and a minimum benefit of q^L in period 2.

This leaves us with:

$$U_i = q^j - p_1 + \mathbb{1}_{\{R_t=1\}}\{[\frac{q^j}{2} - p_1][p_2 - \frac{q^H + q^L}{2}] - e\}$$

Next, we replace $\frac{q^j}{2}$ with κq^j, where $\kappa \in [0,1]$ and $[p_2 - \frac{q^H+q^L}{2}]$ with Δ. This reflects the notion that an equitable payoff may not be one of equal split, allowing us to generalise the equitable point. Hence, when κ is sufficiently high, firms are able to charge some price slightly below quality and still receive a positive rating if e is sufficiently small. We do not make any assumptions over Δ, except that it is positive. This allows us to capture that consumers may not fully understand how firms benefit from ratings, only that a good rating is beneficial for a firm, and a bad rating can harm the firm. Thus, capturing kindness from consumers which enables firms to gain some benefits in subsequent periods.

Finally, we split the consumption utility and the rating utility. This allows for more compatible purchase decision across periods as consumer's purchase decision does not depend on whether they anticipate giving a good rating.

[40] Rabin notes that doing so does not affect the behavior of the utility function.