Competition Through Recommendations

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14 November 2025

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Abstract

This paper examines how two-sided platforms develop their recommender systems to be precise about value-for-money. On each platform, more precise recommendations generate ranking and screening effects: they steer demand toward high value-for-money products, intensifying price competition among firms, and drive out lower-quality firms. Thus, more precise recommendations benefit consumers but reduces platform's per-transaction revenue. A monopolist platform still prefers precise recommendations, as this expands demand. Competing platforms choose even more precise recommendations. However, when consumers search across platforms or recommender systems are overly complex, recommendations become less precise. This shows that market power is only one potential explanation for *ensh*ttification*.

JEL: D21, L10, L86

Keywords: Digital economy, Recommender systems, Two-sided platforms

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This project has especially benefited from early discussions with Paul Belleflamme, Francis Bloch and Bernard Caillaud. Additionally, I thank Philipp Brunner, Christoph Carnehl, Ben Casner, Johannes Johnen, Muxin Li, Martin Peitz, Francisco Poggi, Andrew Rhodes, David Ronayne, Maximilian Schäfer, Nicolas Schutz, Anton Sobolev, Egor Starkov, André Stenzel, Roland Strausz, Greg Taylor, Tat-How Teh, Paul Wegener, Lily Yang and participants of various seminars for their many helpful comments.

I am grateful to have received support from the Fédération Wallonie-Bruxelles (FWB) Action de recherche concertée grant 19/24-101, the Belgian National Fund for Scientific Research (FNRS) Aspirant research fellowship (FC46885), the Deutsche Forschungsgemeinschaft (DFG) through CRC TR 224 (Project B05).

1 Introduction

A key feature of modern online platforms (e.g., search engines, e-commerce websites, social media) is their ability to provide relevant suggestions for users navigating an overwhelming volume of information. Early recommender systems employed simple rules that relied on consumer ratings to recommend products to others. Today, platforms rely on complex (and often opaque) algorithms, coupled with granular data on user behavior to develop these systems. Growing consumer reliance on recommender systems and the rise of opaque data-driven algorithms have elevated concerns about their specification. In response, regulators have introduced measures to protect consumers and promote competition on and between platforms.[1]

This paper explores how two-sided platforms optimally develop their recommender systems, focusing on how competitive pressures between platforms shape the precision of recommendations. I consider a two-sided platform that recommends products to consumers based on value-for-money. Firms produce products of different qualities and face no marginal costs. Firms may choose to enter the platform at no cost, set prices and pay the platform an ad valorem fee. Consumers face some cost of joining a platform and receiving its recommendation.[2] After obtaining the platform's recommendation, consumers purchase a product, which they may return at no cost if the product provides negative consumption utility.

On the platform, firms with higher value-for-money are displayed more prominently, and consumers are more likely to engage with them. The platform may augment its recommender system to be more (or less) precise about value-for-money. A more precise recommender system means consumers are more likely to engage with firms offering a higher value-for-money.

A more precise recommender system causes firms offering higher value-for-money to obtain substantially more demand—a ranking effect. To improve their ranking, firms compete more fiercely on prices. Intensified price competition leads lower quality firms to become unprofitable, having to set negative prices to obtain demand, and exit the market—a screening effect.

Although screening retains only higher-quality firms on the platform, ranking causes prices to fall significantly. As a result, a more precise recommender system leads a platform charging ad valorem fees to earn less per-transaction revenue. However, both ranking and screening work to improve consumer surplus. Ranking results in cheaper products, and both ranking and screening increase the likelihood that consumers engage with higher quality products. Hence, more precise recommender systems enable platforms to attract more consumers, expanding demand.

Using this trade-off between demand expansion and per-transaction revenue, I show that a monopolist platform augments its recommender system to be more precise about value-for-money. This improves consumption utility, attracting more consumers to the platform. The lowest-quality firms exit the market because of the screening effect. Of the firms remaining on the platform, all firms face a negative price effect due to fiercer price competition, and lower quality firms face a negative demand effect as consumers are directed toward firms offering higher value-for-money. Only the highest quality firms experience a positive demand effect that

[1]Examples of such regulators include the Competition and Markets Authority, Cyberspace Administration of China, European Commission, and Federal Trade Commission.

[2]This may be a result of a lack of willingness to join the platform. For example, because they simply prefer not to shop online, have privacy concerns, or resist big corporations.

dominates the price effect, allowing them to obtain higher profits.

Many platforms use complex algorithms when implementing their recommender systems, raising concerns that recommendations may not be well-justified. To understand concerns about recommender system opacity, I consider a model of naive consumers who fail to take into account how firms update prices following the implementation of a precise recommender system. Instead, these consumers believe recommender systems only reflect value-for-money. Here, platforms prefer less precise recommender systems, but also make lower profits than they would if consumers were not naive. Hence, it is in the platform's interest to be voluntarily transparent and educate consumers about its recommender system. This suggests that platforms and regulators may have aligned interests in transparency and consumer education.[3]

In the monopoly setting, I also show how costly production inherently causes firms to set higher prices. This enables a platform taking ad valorem fees to improve its recommender system, thereby favoring consumers. I show that the above results are robust to (i) restricting free returns; (ii) allowing the platform to develop recommender systems less precise than value-for-money and a general distribution of consumer costs; and (iii) a more general class of recommender systems.

When a new platform enters the market and consumers single-home, a unique symmetric equilibrium arises in which both platforms choose more precise recommender systems than a monopolist would. More precise recommender systems improve consumer surplus, which makes the platforms more appealing to consumers. As a result, consumers on the incumbent platform benefit in both the intra- and infra-marginal sense. Existing consumers gain more surplus in expectation, and new consumers are attracted to the platform. Additionally, some consumers with a high cost of joining the incumbent—who would have been inactive in the monopoly setting—may now become active on the entrant. This finding also highlights how competition between platforms can lead to downstream competition between firms within a platform.

However, the algorithms used by these recommender systems require substantial data about products and consumer behavior. Hence, it is less feasible for an entrant to create recommender systems that are as precise as the incumbent. To understand these implications of platform entry, I first consider the extreme environment where the entrant is unable to use recommender systems. Here, the incumbent instead prefers recommender systems that are less precise about value-for-money than a monopolist would. This is because the screening effect leads relatively lower-quality firms on the incumbent platform to migrate to the entrant. However, on the entrant platform, such firms are higher-quality relative to the other firms on the entrant, which increases the expected utility consumers receive there. Hence, by lowering the precision of its recommender system, the incumbent lowers the level of screening, reducing competition for consumers from the entrant.

I also show that as the entrant adopts more precise recommender systems, the incumbent responds by improving its recommender system, benefiting consumers. In this sense, better data access could level the playing field between platforms and serve to improve consumer surplus, lending support for data sharing obligations mandated in the European Union's (EU) Digital

[3]For example, the European Union's (EU) Digital Services Act (DSA) requires platforms to be transparent about the factors taken into account by their recommender systems.

Markets Act (DMA).

I evaluate the role of consumer multi-homing, allowing consumers to multi-home by searching across platforms, this leads to a symmetric equilibrium in which platforms focus on raising per-transaction revenues by worsening recommender systems.

To explore asymmetric equilibria, I consider platforms competing sequentially. Competition causes the incumbent to imp

Additionally, I study two environments of asymmetric competition between platforms. First, I allow consumers to have asymmetric costs. If consumers face a higher cost of joining the entrant, the entrant selects more precise recommender systems than the incumbent. Second, if platforms compete sequentially, competition causes the incumbent to improve its recommender system, but consumers remain worse off than when platforms compete simultaneously.

In the context of sequential platform competition, I evaluate how costly firm entry onto platforms can affect the way firms multi-home. Here I show that only the highest-quality firms multi-home and costly firm entry can raise the expected consumption utility consumers receive from the incumbent compared to costless firm entry. Interestingly, some firms of intermediate quality, which are able to sell on the incumbent, prefer to join the entrant and become 'big fish in a smaller pond'.

Taken together, this paper shows how the competitive environment affects the decision to develop precise recommender systems. The results closely resemble changes in recommender systems on platforms such as Amazon: A new platform with naive consumers prefers a recommender system that purely reflects value-for-money. As consumers learn how the platform's recommender system works, becoming less naive, the platform is incentivized to develop more precise recommender systems. Competition further induces more precise recommender systems. However, when a monopoly is established, recommender systems deteriorate relative to the competitive benchmark. This offers market power as an explanation for platform degradation, sometimes colloquially referred to as *ensh*ttification*.[4] However, as the extensions highlight, there can be numerous other potential explanations for platform degradation.

The rest of the paper is structured as follows: Section 2 describes and analyzes a monopolist platform, with extensions discussed in Section 3. Section 4 introduces competition to the model, and Section 5 shows extensions to this setting. Section 6 provides a discussion of the results. Finally, Section 7 reviews the literature and Section 8 concludes. Proofs can be found in Appendix A and details of some extensions are left to Appendix B.

2 Monopoly

To guide exposition and highlight the trade-offs faced by platforms in isolation, I first consider a monopolist setting. Consider an environment with the following agents: consumers, firms, and a monopolist incumbent platform.

Consumers. There exists a unit mass of consumers each demanding a single unit of product. Consumers have homogeneous preferences for products, but face a heterogeneous cost of joining the platform. This cost is independently and identically drawn from a uniform distribution with

[4]The term *ensh*ttification* was popularized in 2022 by Cory Doctorow to describe the growing difficulties one faces when searching for products on Amazon. The term was named 'word of the year' by the American Dialect Society (2023) and Macquarie Dictionary (2024).

support $c_i \sim U[0,1]$.[5] Consumers choose to join the platform if their expected consumption utility on the platform overcomes the cost of joining it. Upon joining the platform, consumers receive product recommendations and select a particular product to purchase. Consumption utility is the difference between the product's quality and its price, $u(\alpha_j, p_j) = \alpha_j - p_j$, where α_j is the product quality of some firm j and p_j its price.[6] Let n represent the mass of consumers whose cost lower than expected utility, and such consumers join the platform.

Firms. There exists a unit mass of single-product firms. Firms sell products which are substitutes, with heterogeneous quality independently and identically drawn from a continuous uniform distribution, $\alpha_j \sim U[0,1]$ represents the private quality information drawn by firm j. Firms may only sell through the platform. In other words, there is no direct channel of sales. Firms face zero marginal cost of production and no entry costs. However, they pay a commission fee, $r \in (0,1)$, on revenue to the platform. Each firm selects a price p_j to maximize its profits, $\pi(D_j(\lambda, \alpha_j, p_j, \mathbf{p}_{\text{-}j}), p_j) = D_j(\lambda, \alpha_j, p_j, \mathbf{p}_{\text{-}j})(1-r)p_j$ where $D_j(\lambda, \alpha_j, p_j, \mathbf{p}_{\text{-}j})$ is the demand firm j, with quality α_j and setting price p_j, obtains given prices of all other firms on the platform, represented by the vector $\mathbf{p}_{\text{-}j}$, and λ represents the recommender system adopted by the platform. Firms are active if they make weakly positive profits, and where they make zero profits, a firm prefers to sell rather than not. Therefore, a firm makes two decisions: the choice of joining the platform and its selling price. Let $\mathbf{N}$ represent the set of firms joining the platform.

Platform. The incumbent is a monopolist platform acting as an intermediary between firms and consumers. The platform earns revenue from an exogenously determined ad valorem commission fee charged to firms, $r \in (0,1)$.[7] Hence, the platform's profits are $\Pi = r\int_{j\in\mathbf{N}} D_j(\lambda, \alpha_j, p_j, \mathbf{p}_{\text{-}j})p_j \; d\alpha_j$. The platform may learn the quality of firms and provide recommendations through a listing to consumers, following some recommender system.[8] Let the recommender function $\lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma)$ be the probability with which consumers interact with the listing of a particular firm j, where σ is determined by the platform and governs the shape of the probability density function. To aid exposition, I suppose $\sigma \in \mathbb{R}_+$, and in Appendix B, I show that results are robust to $\sigma \in \mathbb{R}$.

Hence, the demand firm j faces can be written as $D_j(\lambda, \alpha_j, p_j, \mathbf{p}_{\text{-}j}) = n \times \lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma)$, where the mass of consumers joining the platform are those whose expected consumption utility exceeds their cost, $E[u] > c_i$, such that $n = E[u]$ and

$$E[u] = \int_{j\in\mathbf{N}} \lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma)(\alpha_j - p_j) \; d\alpha_j. \tag{1}$$

Note that consumers' expected utility depends on the set of firms that choose to be active on

[5] I relax this assumption in Appendix B, allowing for more general distributions of c_i.

[6] The cost of joining a platform does not feature in consumption utility as consumption occurs after consumers join the platform. In other words, at the point of making the consumption decision, joining is a sunk cost.

[7] For example, historically, eBay charges a nominal listing fee and charges a commission which is a percentage of the final transaction value. In 1998, the listing fee was between 25 cents and 2 dollars and the commission fee between 1.25% and 5% of final transaction value (eBay, 1998). In 2005, The New York Times (2005) reported the commission fee would increase from 5.25% to 8%.

[8] In practice, platforms are not able to directly observe firm quality. Instead, they learn about firm quality by aggregating feedback from consumers. Hence, it is not possible for platforms to simply direct consumers to the 'best' firm as, without feedback, the platform is unable to identify such a firm.

the platform—that is, firms which can make a positive profit.

The firms' and platform's profits respectively evaluate to

$$\pi(\lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma), p_j) = n\lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma)(1-r)p_j \tag{2}$$

$$\Pi = nr \int_{j\in\mathbf{N}} \lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma)p_j \; d\alpha_j. \tag{3}$$

In practice, firms higher on a list are more likely to receive interactions from consumers. To reflect this, I model recommender systems following a Tullock contest (Tullock, 1980):

$$\lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma) = \begin{cases} \frac{\alpha_j - p_j - \sigma}{\int_{h\in\mathbf{N}} \alpha_h - p_h - \sigma \; d\alpha_h} & \text{if } \alpha_j - p_j - \sigma \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

To understand this contest success function, first fix $\sigma = 0$. Then firms providing a higher value-for-money obtain relatively more demand. When $\sigma < 0$, this introduces noise into the system, and demand is more evenly distributed—firms providing a relatively higher value-for-money no longer receive as much more demand as before. Conversely, when $\sigma > 0$, this makes the system more precise about value-for-money, this means the platform steers consumers towards firms providing a higher value-for-money and these firms now receive significantly more demand than those with lower value-for-money.

This modified Tullock contest assumes that consumers receive positive surplus from a transaction. In Appendix B, I relax this assumption and allow consumers to be recommended and consume products that provide negative consumption utility, and also show that the main findings apply to a class of more general recommender functions that recommend firms to consumers based on relative consumption utility. In either case, the qualitative insights of the model survive.[9] Appendix B also provides some search-based microfoundations for direct application of a contest success function to determine how consumers interact with firms.

This recommender system is reminiscent of how consumers do not simply observe a single recommendation when searching on platforms. Instead, consumers observe a list of products, and are more likely to interact with products featured more prominently on the list. Moreover, when $\sigma = 0$, platforms provide exactly value-for-money recommendations. This is of particular interest because platforms initially, and can easily and directly, rely on consumer ratings, reviews, and feedback to populate their recommendations. Prior work has also shown such consumer-generated feedback reflects value-for-money (Li and Hitt, 2010; Cai et al., 2014; Neumann et al., 2018; Luca and Reshef, 2021; Carnehl et al., 2025). In other words, a platform can indirectly learn about the quality of the firm only after some transactions have occurred and, in equilibrium, ranks firms with higher value-for-money higher on the list.

Additionally, mature platforms, and those with the ability to invest in other recommendation tools may offer a recommender system different from value-for-money. Instead, their recommendations may be more precise about consumption utility through the use of tools such as sophisticated recommendation scoring rules (Amazon, 2024; Xu, 2022), verification processes

[9] I also assume that platforms can select σ costlessly. This assumption is innocuous and all qualitative effects would survive if there was some cost $S(\sigma)$ where $S'(\sigma) \geq 0$. The key difference is that the optimal level of σ would be lower, and in the asymmetric competition setting more flat across platforms.

(Google, 2024; Tripadvisor, 2024), 'badges' and awards (Booking, 2024; Airbnb, 2024), buy-boxes, and limited time deals. These recommendation tools, still based on value-for-money, allow platforms to effectively skew recommendations in favor of products that consumers prefer more, $\sigma > 0$ represents this outcome. In other words, higher levels of σ reflect recommendations which are more precise about consumption utility, and in practice, consumers, recognizing such recommendations and anticipating a higher value-for-money, select them with a higher probability.

In addition to highlighting the effects of the model, studying a monopolist platform reflects the status of Very Large Online Platforms (VLOPs), defined and regulated by the EU's DSA, and Gatekeepers, defined and regulated by the EU's DMA, and their role as key intermediaries between firms and consumers. Some examples include Amazon Store, Google Play, Google Maps, Google Shopping, Google Search, Alibaba AliExpress, Meta Facebook, and Apple App Store. Importantly, Article 27 of the DSA expressly targets recommender systems, requiring enhanced transparency and user agency.

The sequence of events follows: (1) firms learn their quality and the platform announces σ; (2) firms decide to join the platform and the platform learns the quality of firms which join; (3) firms joining the platform set prices; (4) consumers decide to join the platform and make a consumption decision. I solve for a subgame perfect Nash equilibrium.

2.1 No recommender system

I first construct a benchmark in which the incumbent does not implement a recommender system and consumers engage with firms at random. In other words,

$$\lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}) = \frac{1}{\int_{h \in \mathbf{N}} 1 \; d\alpha_h} .^{10}$$

This is reminiscent of the early days of e-commerce on the internet (e.g. eBay) prior to the implementation of seller feedback systems. Platforms had little means of identifying firm characteristics and, by extension, were unable to provide recommendations to consumers. I search for prices that maximize consumer surplus—the difference between consumption utility and joining cost, providing the most optimistic consumer-centric outcome without recommender systems. In equilibrium, the consumer-optimal price levels are $p_j = 0$ and the total welfare and consumer surplus are $1/8$.[11]

2.2 Value-for-money recommendations

As more transactions occur on a platform, it is able to gather feedback about firms from consumer purchase behavior and from ratings and reviews.[12] This allows them to learn the value-for-money that products provide and, in turn, list products with higher value-for-money more prominently than those with lower value-for-money. Hence, it is natural to consider a recommender system which captures value-for-money.

[10] In a slight abuse of notation, I impose that this is equivalent to setting $\sigma = -\infty$.

[11] Details can be found in Appendix B.

[12] While dynamic collection and use of transaction records and consumer feedback (ratings and reviews) is not explicitly modeled, platforms can use such data to approximate the value-for-money of products (Li and Hitt, 2010; Gutt and Herrmann, 2015; Neumann et al., 2018; Luca and Reshef, 2021).

Illustrating recommender systems that purely reflect value-for-money also serves to highlight the ranking effect introduced by recommender systems. Constraining $\sigma = 0$:

$$\lambda^v(\alpha_j, p_j, \mathbf{p}_{\text{-}j}) = \begin{cases} \frac{\alpha_j - p_j}{\int_{h\in\mathbf{N}} \alpha_h - p_h \; d\alpha_h} & \text{if } \alpha_j - p_j \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Because $\alpha_j \geq p_j$, consumers always purchase if they join the platform, and the mass of consumers is given by

$$n^v = E[u^v] = \int_{g\in\mathbf{N}} \frac{\alpha_g - p_g}{\int_{h\in\mathbf{N}} \alpha_h - p_h \; d\alpha_h} (\alpha_g - p_g) \; d\alpha_g.$$

Each firm's profit function evaluates to

$$\pi(\lambda^v(\alpha_j, p_j, \mathbf{p}_{\text{-}j}), p_j) = n^v \frac{\alpha_j - p_j}{\int_{h\in\mathbf{N}} \alpha_h - p_h \; d\alpha_h} (1-r) p_j.$$

Notice that each firm is unable to unilaterally influence n^v, and firm j maximizes its profits by setting the optimal price $p_j^v = \alpha_j/2$.[13] Accounting for prices,

$$\lambda^v(\alpha_j, p_j^v, \mathbf{p}_{\text{-}j}) = \begin{cases} \frac{\alpha_j}{\int_{h\in\mathbf{N}} \alpha_h \; d\alpha_h} & \text{if } \alpha_j \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

which shows how value-for-money recommender systems create a *ranking effect*, in which higher-quality firms are now able to capture a larger share of demand relative to lower-quality firms.

To understand the market effects, it is straightforward to compute the profits and consumer surplus. The platform and firms share the total profit 1/9, while consumer surplus is 1/18. Hence, comparing value-for-money recommender systems to the consumer surplus-optimal outcome with no recommender system:

Remark 1. *When implementing value-for-money recommendations:*

1. *Consumer surplus falls and fewer consumers join the market.*
2. *Total welfare increases.*

Although consumers may purchase higher-quality products more often, due to the increase in price, they receive less surplus and are less likely to join the platform.

2.3 Precise recommendations

While the previous two subsections ignore the platform's strategy, they closely capture historical events. Now consider a platform being able to specify a recommender system which emphasizes value-for-money, by choosing σ to maximize its profits.

Consumers join the platform if their expected consumption utility exceeds the cost of doing so, and $\alpha_j - p_j - \sigma \geq 0$ implies $\alpha_j - p_j \geq 0$ such that they always purchase if they join the platform. The mass of consumers evaluates to $n^m = \int_{g\in\mathbf{N}} \frac{\alpha_g - p_g - \sigma}{\int_{h\in\mathbf{N}} \alpha_h - p_h - \sigma \; d\alpha_h} (\alpha_g - p_g) \; d\alpha_g$.

[13] If the firm faces some costs (e.g. marginal cost of production) qualitatively similar effects apply, see Section 3.

Firms are unable to unilaterally influence consumers' entry decision, and each firm j maximizes its profit

$$\pi(\lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma), p_j) = n^m \frac{\alpha_j - p_j - \sigma}{\int_{h\in\mathbf{N}} \alpha_h - p_h - \sigma \; d\alpha_h}(1-r)p_j$$

setting the optimal price $p_j^* = \frac{\alpha_j - \sigma}{2}$. More precise recommendations (larger σ) lead to fierce price competition, which leads firms to set lower prices. Higher quality firms set higher prices than lower quality firms. Accounting for prices,

$$\lambda(\alpha_j, p_j^*, \mathbf{p}_{\text{-}j}, \sigma) = \begin{cases} \frac{\alpha_j - \sigma}{\int_{h\in\mathbf{N}} \alpha_h - \sigma \; d\alpha_h} & \text{if } \alpha_j - \sigma \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

This formulation shows how precise recommendations further highlights the ranking effect as the relative likelihood of obtaining some demand increases for higher quality firms and decreases for lower quality firms. As a result, higher quality firms are more profitable than lower quality firms as they benefit from higher prices and receive more demand.

Additionally, precise recommendations (larger σ) introduce a screening effect where low quality firms receive zero demand. Because precise recommendations mean firms have to provide higher value-for-money to consumers to obtain demand, such low quality firms have to set much lower prices. However, if a firm has to set prices below marginal cost to attract demand, it would rather become inactive. In other words, a firm is only active if $\alpha_j \geq \sigma$. The resulting ranking and screening effects are illustrated by Figure 1, and Lemma 1 summarizes findings about firms.

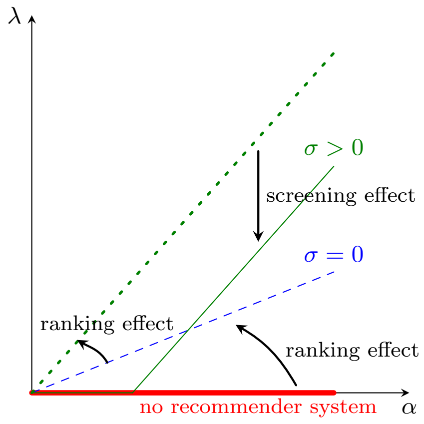


Figure 1: When no recommender system is used, all firms have equal probability (zero mass) of being selected by consumers (thick red line). When $\sigma = 0$, a ranking effect, a rotation, is introduced (dashed blue line). When $\sigma > 0$, the ranking effect is more pronounced (dotted green line) and an additional screening effect, a translation, is introduced (solid green line).

Lemma 1.

- *Under precise recommendations, relatively higher quality firms make more profit than relatively lower quality firms.*
- *When $\sigma > 0$, only firms with sufficiently high quality, $\alpha_j > \sigma$, are active on the incumbent.*
- *When recommendations are excessively precise, $\sigma \geq 1$, all firms become inactive.*

Notice that the outside option for firms is zero as there is no direct channel of sales. Hence, as an implication of Lemma 1, firms with quality lower than σ exit the market. This provides the following Corollary:

Corollary 1. *There exists a cutoff firm, $\bar{\alpha} = \sigma$, above which all firms are active on the incumbent platform, and below which all firms are inactive.*

Finally, consider the incumbent's problem. Given consumer entry, firm pricing decisions and $\bar{\alpha} = \sigma$, the platform's profit function (3) evaluates to

$$\Pi^{=}n^m r \int_{\sigma}^{1} \lambda(\alpha_h, p_h^*, \mathbf{p}_{\text{-}h}, \sigma) p_h^* \, d\alpha_h = \frac{1+2\sigma}{3} r \frac{1-\sigma}{3}.$$

The platform optimizes its recommender system by balancing demand for the platform and the revenue it is able to extract from firms. On the one hand, more precise recommendations lead to demand expansion in two ways: First, consumers have a higher probability of transacting with higher quality firms. Second, firms set lower prices which provides consumers with a larger share of the transaction surplus. On the other hand, per-transaction revenue suffers as firms lower prices.

In equilibrium, Proposition 1 shows the platform prefers recommendations which are more precise about consumption utility than recommendations that are purely based on value-for-money.

Proposition 1. *There exists a unique subgame perfect Nash equilibrium where the incumbent platform sets $\sigma^m = 1/4$, developing recommender systems which are more precise than value-for-money recommendations.*

2.4 Welfare discussion

More precise recommendations make consumers better off as they are more likely to purchase from a higher quality firm and, given quality, pay lower prices. Hence, consumers are better off when a platform adopts precise recommendations rather than value-for-money recommendations. Additionally, when comparing precise recommendations to no recommender system, despite paying higher prices consumers can be just as well off as no recommender system. This is because they transact with higher quality firms more often, allowing them to obtain an on average higher surplus. Therefore, in equilibrium, consumers are indifferent between using a monopolist platform with precise recommendations over one with no recommender system. Corollary 2 summarizes these results.

Corollary 2. *On the monopolist, consumer surplus increases when recommendations are more precise ($\sigma \uparrow$). In equilibrium, consumer surplus on the monopolist with precise recommendations is* $1/8$.

Recall also that Section 2.1 makes the strong assumption that prices are consumer-optimal. Relaxing this assumption would imply that a monopolist platform with precise recommendations strictly benefits consumers.

Computing profits, the platform makes $\Pi^m = r/8$ and the total profit firms make is $(1-r)/8$. Although aggregate firm profit is larger than value-for-money recommendations ($\sigma = 0$), not all firms benefit from precise recommendations and those which do, do so unequally. For intuition, recall from Corollary 1 that some lowest quality firms are screened off the market. Hence, these firms become inactive and cannot benefit from precise recommendations. The remaining firms see precise recommendations affect their demand through two channels: (i) the entry of infra-marginal consumers, benefiting all active firms, (ii) a redistribution of demand from lower to higher quality firms benefits (harms) higher (lower) quality firms. Moreover, more precise recommendations lead to fiercer price competition and all active firms set lower prices. Hence, for some highest quality firms the demand effects dominate the price effect but for the remaining firms, the benefits from the introduction of infra-marginal consumers is unable to dominate both the negative redistribution and price effects. Corollary 3 formalizes this.

Corollary 3. *When recommendations are more precise ($\sigma \uparrow$), firms with quality above some cutoff, $\hat{\alpha}$, receive higher profits, while all other firms receive less profit. This cutoff is increasing in σ.*

Although consumers are indifferent between a monopolist platform with a precise recommender system and no recommender system, because the total profit of the platform and firms is 1/8, precise recommender systems can improve total welfare.

User agency A monopolist platform thus has an incentive to use precise recommendations. Since precise recommender systems improve total welfare and makes consumers at least as well off as no recommender system, this shows a platform providing a recommendation service is able to generate value in the market. Importantly, if users are left to their own devices (to select products based solely on consumer feedback or not use recommender systems), consumers are weakly worse off than following the platform's recommendation (Corollary 2). This suggests that Article 27 of the DSA requiring platforms to provide consumers with the ability to modify the main parameters of their recommender systems cannot improve consumer surplus and hence consumers are unlikely to rely on such features when interacting with recommender systems. In other words, it is likely that such regulation imposes a superficial and costly requirement onto the way recommender systems are developed while having no (or negative) effect on both consumer surplus and welfare.

3 Monopoly extensions

3.1 Naive consumers

The main analysis assumes consumers fully understand the equilibrium implications of how platforms develop recommender systems. However, consumers may not fully understand the implications of recommender systems. For example, due to a lack of exposure, or because of the increasingly complex nature of these systems.

This section considers consumers who are naive and unable to fully appreciate the implications of more precise recommender systems. One possible scenario is that consumers do not correctly anticipate how firms adjust prices in response to the platform's recommender system. To capture this, I model naive consumers who fail to recognize how firms account for σ in their pricing strategy, even when firms actually do.

Because naive consumers do not anticipate how firms reduce prices in response to more precise recommender systems, they believe the effect of more precise recommender systems on their consumption utility is smaller than it actually is. Hence, their decision to join the platform is less responsive to σ. Intuitively, the platform responds by lowering σ. If precise recommender systems are less likely to attract consumers, then the platform can focus on obtaining higher per-transaction revenues by emphasizing value-for-money less (lower σ) which reduces price competition between firms. In equilibrium, the platform sets $\sigma^N = 0$.[14] Since the platform earns lower profits as it deviates from $\sigma^m = 1/4$, it makes less profit when consumers are naive. Moreover, from Corollary 2, we know consumers are worse off when they are naive.

Transparency and public education These results then imply that consumer education and algorithmic transparency are necessary to improve market outcomes and such policies would be supported both by regulators and platforms.

For regulators, the result supports regulatory concerns that lack of transparency about how complex recommender systems work can lead to worse recommendations. These concerns are validated as consumers can be worse off than with no recommender systems. If regulations such as Article 27 of the DSA—which requires transparency of recommender systems to consumers—can make the implications of recommender systems, in particular how their specification may affect firms' prices and entry decisions, more salient to consumers, this can reduce naïveté, which in turn leads to the development of more precise recommender systems and improves consumer surplus. For the platform, it makes larger profits when consumers are not naive, which suggests that if the cost of developing more precise recommender systems is sufficiently low, platforms too prefer to educate consumers about their recommender systems. Hence, when it comes to recommender systems, although platforms and regulators may face different goals, it is possible the act of transparency and consumer education works towards achieving their respective goals.

For details, see Appendix B.

3.2 Marginal costs

The main analysis assumes firms face zero marginal cost of production. Here, I relax this assumption and suppose all firms face a marginal cost of production e. Proposition 2 shows that the platform prefers more precise recommendations when firms face a higher marginal cost. Note that to cover marginal costs, at a given σ, any firm of quality α_j has to set higher prices. This has two effects. First, some lower quality firms become inactive as they are unable to both cover the increase in marginal cost and induce positive demand from consumers following (4). Second, because the platform charges an ad valorem fee, when firms raise prices the platform's per-transaction revenue increases. Both effects serve to improve the platform's per-transaction revenue. Since prices are now less elastic in recommender system precision, the platform focuses on its other trade-off: developing more precise recommender systems to attract more consumers.

Proposition 2. *A monopolist platform develops more precise recommender systems when firms face higher marginal cost of production,* $\frac{\partial \sigma}{\partial e} > 0$.

This suggests that lower production costs are a contributing factor to platform degradation.

[14] It is also possible to consider that only a share of users are naive. Then the platform prefers some intermediate level of precision, $\sigma \in (\sigma^N, \sigma^m)$. Details can be found in Appendix B.

If firms face a lower production cost, then the platform has an incentive to develop less precise recommendations to raise prices.

3.3 Monopoly extensions

In Appendix B, I show that the platform's trade-offs and welfare effects are robust to: (i) *no free returns* such that consumers may end up purchasing, and keeping, a product which provides negative consumption utility. (ii) *Uninformative recommendations and distributional assumptions.* I simultaneously allow $\sigma \in \mathbb{R}$ such that a platform can also specify recommender systems which are more noisy about consumption utility and let c_i be drawn from a generic distribution with support $[0, 1]$ and peak 1. (iii) *General recommender functions* by adopting more general contest success function that depends on relative consumption utility.[15] For details, see Appendix B.

4 Competition

While most commonly discussed online platforms seem like monopolists in their own markets, they often face competition and coexist with smaller, lesser-known platforms. For example, Amazon competes with Newegg in the computer hardware market and more generally with big-box stores and discounters such as Walmart and Zalando.

In this section, I explore the situation where two platforms, $k \in \{I, E\}$—the incumbent and entrant respectively—compete simultaneously in their recommender systems. Firms may choose to multi-home and joining each platform is costless for the firm. Consumers' cost to join each platform, $c_{i,k}$, is independently and identically drawn from a uniform distribution with support $[0, 1]$. Consumers do not multi-home and join the platform which provides the highest expected surplus—expected consumption utility less cost of joining the platform, $E[u_k] - c_{i,k}$. Hence, the mass of consumers joining each platform k is

$$n_k = \begin{cases} E[u_k] - \frac{E[u_{\text{-}k}]^2}{2} & \text{if } E[u_k] \geq E[u_{\text{-}k}] \\ E[u_k](1 - E[u_{\text{-}k}] + \frac{E[u_k]}{2}) & \text{if } E[u_k] < E[u_{\text{-}k}] \end{cases} \tag{5}$$

where the expected consumption utility from joining either platform follows (1). Let the subscript $k \in \{I, E\}$ represent the decisions on each platform k.

To provide a benchmark for the model with competition, I first consider an entrant with no recommender system and later allow the entrant to use precise recommendations.

4.1 Entrant: no recommender system

Studying an entrant with no recommender system reflects the situation where new platforms may be unable to provide recommendations, for example, due to a lack of technology or data. Using this as a benchmark also draws attention to a key dynamic faced by the incumbent when developing its own recommender system: To prevent firms from joining the entrant, the incumbent chooses to provide a less precise recommender system.

Following Section 2.1, suppose that when a platform has no recommender system, prices on the platform are consumer surplus optimal. Therefore, the price that a firm with quality α_j

[15] I also discuss two specific contest success functions: Tullock contest with utility exponent and logistic contest success function.

sets on the entrant is $p_{j,E} = 0$.

On the incumbent platform, firms are unable to unilaterally influence consumers' arrival to the platform. Hence, firms joining the incumbent adopt the same price strategy as on a monopolist platform, $p^*_{j,I} = \frac{\alpha_j - \sigma_I}{2}$. Because firms joining the entrant make zero profit, firms compare being on the incumbent platform to their outside option of joining the entrant, zero. Therefore, the strategies of firms are identical to those in Section 2.3 and Lemma 1 applies to firms choosing to join the incumbent in this setting.

Although firms can multi-home, Corollary 4 shows that they prefer to single-home, with higher quality firms being active on the incumbent and lower quality firms active on the entrant. Intuitively, all firms prefer being on the incumbent, as this allows them to make positive profits. When a firm is active on the incumbent, it does not multi-home. Supposing a positive mass of firms choose to multi-home. This raises the expected consumption utility that consumers receive from joining the entrant, which draws consumers away from the incumbent; see (5) and Figure 2c for illustration. Hence, by joining the entrant and making zero profits, the firm may improve its total demand, but cannibalizes profit from itself by attracting consumers towards the entrant. However, if a firm is screened off from the incumbent, because firms prefer selling to not selling, it joins and sells on the entrant.

Corollary 4. *There exists a cutoff firm $\bar{\alpha} = \sigma_I$, above which all firms are only active on the incumbent, and at and below which all firms are only active on the entrant.*

Corollary 4 is analogous to Corollary 1. The key distinction between Corollary 1 and Corollary 4 is that firms now have the choice to be active on the entrant, continuing to sell but making zero profits. Applying this result, the expected consumption utility on the incumbent and the entrant are

$$E[u_I] = \frac{1 + 2\sigma_I}{3} \qquad\qquad E[u_E] = \frac{\sigma_I}{2}.$$

Since $\sigma_I \in [0, 1)$ such that $E[u_I] > E[u_E]$,[16] the demand on either platform is

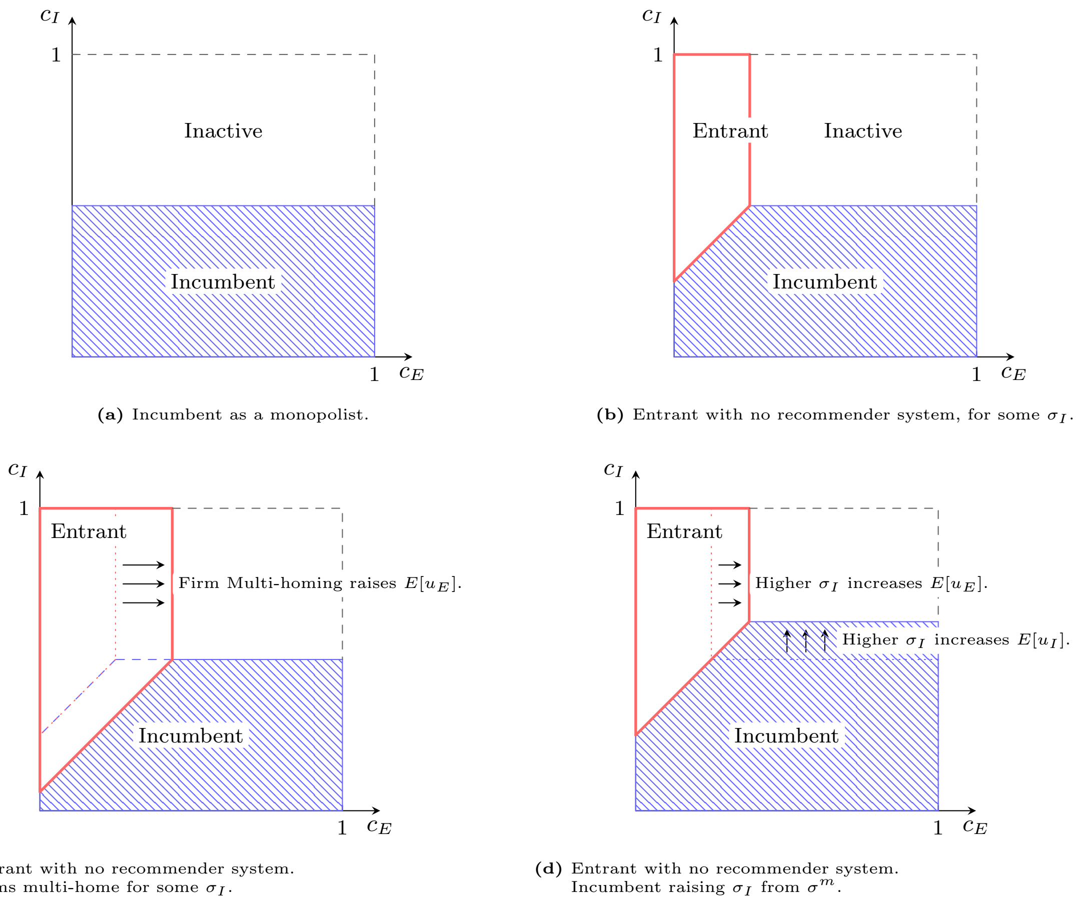
$$n_I = \frac{1 + 2\sigma_I}{3} - \frac{\sigma_I^2}{8} \qquad\qquad n_E = \frac{\sigma_I}{2}(1 - \frac{1 + 2\sigma_I}{3} + \frac{\sigma_I}{4}).$$

Figure 2b illustrates consumers choice of platform following entry.

The presence of the entrant distorts the incumbent's incentive to provide precise recommendations. When the incumbent provides more precise recommendations, it screens a positive measure of relatively lower quality firms off the platform. However, these firms are of relatively higher quality compared to firms on the entrant. Because firms prefer to be active rather than not, they join the entrant and improve the expected consumption utility that consumers receive from joining the entrant. Hence, the incumbent improving its recommender system is only able to attract a smaller mass of new consumers by than without the entrant. This is illustrated in Figure 2d.

However, a platform balances this gain in consumers with the per-transaction revenue it receives. Although more consumers join the platform, firms set lower prices and the aggregate

[16] If $\sigma_I \geq 1$ all firms join the entrant.



(a) Incumbent as a monopolist.

(b) Entrant with no recommender system, for some σ_I.

(c) Entrant with no recommender system. Firms multi-home for some σ_I.

(d) Entrant with no recommender system. Incumbent raising σ_I from σ^m.

Figure 2: Consumers' choice of platform: Consumers in the red dotted region join the entrant. Consumers in the blue shaded region join the incumbent. The remaining consumers are inactive and do not join any platform.

per-transaction revenue falls. This decrease in revenue dominates any gain in transaction volume. As a result, an incumbent considering consumers' incentives to switch between platforms prefers a less precise recommender system than a monopolist. This is expressed in Proposition 3.

Proposition 3. *There exists a unique subgame perfect Nash equilibrium where the incumbent facing an entrant with no recommender system adopts* $\sigma_I = 2/9$.

Notice that $\sigma_I < \sigma^m$. Hence, when facing an entrant with no recommender system, an incumbent makes its recommender system less precise.

Corollary 5 shows that competition between an incumbent employing precise recommender systems and an entrant without recommender systems can improve consumer surplus. However, these improvements in consumer surplus are driven by infra-marginal consumers becoming active in the market.

To see this, recall that the expected consumption utility on the incumbent is $\frac{1+2\sigma_I}{3}$, which is increasing in σ_I. Hence, expected consumption utility on the incumbent is lower in the face of competition. This is a result of both increasing prices and lower firm quality on the incumbent platform. As a result, taking into account consumers' cost of entry, Corollary 5 shows that consumers on the incumbent are worse off and, therefore, gains in total consumer surplus are due to consumers who would otherwise be inactive becoming active on the entrant.

Importantly, this highlights how competition alone is insufficient to incentivize incumbent platforms to foster trust by improving the specification of their recommender systems, and can instead result in platform degradation.

Corollary 5. *Compared to the case where the incumbent is a monopolist, competition with an entrant with no recommender system leads to:*

- *An increase in total consumer surplus.*
- *Lower consumer surplus on the incumbent.*

4.2 Entrant: precise recommendations

To fully understand how competing platforms develop their recommender systems, I now turn to the situation where the entrant also adopts a precise recommender system, (4). Allow the incumbent and entrant to simultaneously decide σ_I and σ_E respectively.

In equilibrium, the mass of consumers joining platform k is given by (5) and consumers always purchase following entry. Since individual firms are unable to influence consumers' platform choice, on each platform all firms adopt the same price strategy as Section 2.3, $p^*_{j,k} = \frac{\alpha_j - \sigma_k}{2}$. Further, all firms with sufficiently high quality, $\alpha_j > \sigma_k$ are able to make a positive profit on the platform k. Since multi-homing is costless, and firms can make a positive profit if their quality is sufficiently high, analogous to Corollary 1, any firm with $\alpha_j > \max\{\sigma_I, \sigma_E\}$ is active on both platforms.

Proposition 4. *There exists a unique symmetric equilibrium where platforms adopt $\sigma_I = \sigma_E = \sigma^s \equiv \frac{5-3\sqrt{2}}{2}$ and each make the profit $\Pi^s < \Pi^m$.*

When the platforms simultaneously select σ_k, the problem is symmetric. Hence, there exists a symmetric equilibrium, in which both platforms adopt more precise recommender systems than a monopolist ($\sigma^s > \sigma^m$).

Contrasting this equilibrium to an entrant with no recommender system, the incumbent no longer has an incentive to reduce recommender systems below the monopolist level. This is because firms may now make positive profits on the entrant. Being able to do so removes the incentives for firms to single-home. When firms multi-home, the incumbent is no longer able to lower the entrant's aggregate quality by strategically allowing lower quality firms onto the incumbent platform.

Compared to a monopolist, this means competing platforms lead to firms on both platforms setting lower prices, only higher quality firms participate in the market and highest quality firms are recommended to consumers more often. Hence, competition between precise recommender systems can improve the surplus of both infra-marginal and intra-marginal consumers.

Notably, competing platforms lead to more precise recommendations, which cause firms on each platform to compete more fiercely in prices. This highlights the importance of considering potential downstream effects of encouraging competition between platforms.

The following remark establishes how an entrant with limited capacity to develop precise recommender systems affects the incumbent's recommender system.

Remark 2. *Suppose the entrant is unable to develop recommender systems as precise as the incumbent, e.g. constrain $\sigma_I > \sigma_E$. Then $\frac{\partial \sigma_I}{\partial \sigma_E} > 0$. The presence of an entrant adopting* *recommender systems more precise of value-for-money improves the precision of the incumbent's recommender system.*

This means, competition is only effective at improving the incumbent's recommender system only if the entrant has the ability to develop sufficiently precise recommender systems. Otherwise, the incumbent specifies worse recommender systems than it would as a monopolist.

Data sharing obligations. A practical question arises: Can an entrant develop a recommender system as precise as the incumbent? Indeed, to build precise recommender systems, platforms require past consumer transactions and feedback. New entrants to a market are unlikely to easily amass such data. Data sharing obligations, such as those imposed by the DMA on Gatekeepers, could mitigate this issue. If individual firms are able to take past transaction information to other platforms, this can reduce their switching cost as they bring along reputation and customer feedback which can be used to develop an entrant's recommender system. As a result, firms can obtain similar profits on the new platform, encouraging multi-homing. Likewise, if consumers can easily transfer past transaction information to a new platform, this reduces switching costs. This way, data sharing obligations potentially allow entrants to develop precise recommender systems similar to those of incumbents.

5 Competition extensions

I consider the following modifications in turn. First, I allow consumers to multi-home across platforms. The remaining extensions address platform asymmetries. Second, I consider platforms that compete in a sequential fashion rather than simultaneously, and I also examine the role of costly firm entry. Third, I allow consumers to face asymmetric costs of joining each platform.

5.1 Multi-homing consumers

A natural extension is to consider the implications of consumer multi-homing by searching across platforms. For example, a consumer may arrive at the first platform and obtain its recommendation, and compare this with a recommendation from a second platform.

Hence, from the previous model, this section relaxes the assumption that consumers cannot multi-home, and allows consumers to search between platforms. The game follows: Platforms simultaneously decide on the specification of their recommender systems. Firms then choose which platform to join and set prices. Following which, consumers choose to join a platform and realize the platform's recommendation. Consumers may then choose to buy immediately or to visit the second platform. If consumers visit the second platform, they learn of that platform's recommendation. Consumers have perfect recall. I solve for a symmetric equilibrium where $\sigma_I = \sigma_E = \sigma^h$.

Proposition 5. *There exists a unique symmetric equilibrium where platforms prefer a less precise recommender system than a monopolist,* $\sigma^h < \sigma^m$.

In equilibrium, consumer behavior is similar to a search model with recall. Consumers first join the platform that provides the highest expected consumption utility less cost of joining. Consumers who simultaneously obtain a poor recommendation and face a low cost of joining the second platform would then choose to search the second platform. All other consumers purchase immediately following the recommendation from the first platform. Consumers who searched both platforms evaluate the products and purchase the product providing the highest consumption utility.

It is intuitive to see why platforms prefer less precise recommendations. Each platform loses market power over consumers because some consumers would simultaneously receive both a poor recommendation and have a very low cost of joining the second platform. This makes retaining consumers more difficult for the platforms. Even if a platform develops more precise recommender systems, it will always lose some consumers to search. Hence, consumer multi-homing makes it more difficult for platforms to improve transaction volume. A platform then favors the other trade-off, improving per-transaction revenues, and does so by developing a less precise recommender system.

This provides a surprising hypothesis: for the same product, consumers' multi-homing behavior would lead to an increase in prices. This hypothesis could be tested using a combination of clickstream data to measure consumer multi-homing behavior and tendencies, and prices of identical products on competing platforms. If an increase in price can be attributed to higher consumer multi-homing, this might suggest that the lack of consumer loyalty worsens recommender system precision.

Consumer loyalty Results from this section suggest a potential channel of platform degradation results from consumer search behavior. If consumers find it easy to compare between platforms, platforms have less incentive to develop precise recommender systems. This means, encouraging consumer search can, surprisingly, lower consumer surplus. Hence, cautious consideration of overall market effects when considering regulations that may encourage consumer multi-homing or reduced customer loyalty.

5.2 Sequential equilibrium

From the model of competition where consumers single-home, suppose that the entrant specifies its recommender system before the incumbent. For example, the incumbent may have more flexibility or market power, allowing it to develop its recommender system in response to the entrant.[17] Hence, the timing of the game follows: The entrant selects σ_E; the incumbent selects σ_I; firms decide which platform to join and set their prices on each platform; consumers choose which platform to join and make their consumption decision.

In this setting, I show that the incumbent prefers a more precise recommender system than the entrant, however, this is strictly lower than the symmetric equilibrium. This suggests that any entrant that is able to develop precise recommender systems would, in equilibrium, lead to better recommender systems than a monopolist.

Importantly, recall that consumer surplus is increasing in the precision of the recommender systems on both platforms. Hence, when an entrant can provide precise recommender systems and consumers single-home, consumers are better off with competition than without. Comparing the sequential and simultaneous equilibria, consumers are worse off while the platforms make more profits if they participate in a sequential manner than simultaneously.

[17]This is without loss, one can alternatively choose to assign the entrant to be more profitable, or have more capacity for flexibility, or the ability to develop better recommender systems, and hence move second.

Proposition 6. *There exists a unique sequential equilibrium where* $\sigma^m < \sigma_E < \sigma_I < \sigma^s$ *and the platforms make* $\Pi_I > \Pi_E > \Pi^s$.

Because it is costless to multi-home, and firms are able to set positive prices on both platforms, all firms which are able to be active on a platform choose to do so. This means that firms with quality $\alpha \geq \sigma_I$ multi-home across both platforms, and those with quality $\alpha \in [\sigma_E, \sigma_I)$ single-home on the entrant.

To further understand firm multi-homing behavior, I now consider the effects of costly firm entry.

Costly firm entry As an extension to the model with sequential platform competition, suppose firms are able to multi-home. For firms, joining the first platform is free but joining the second platform is costly. As above, without loss of generality, allow the entrant platform to move first and the incumbent follows. Proposition 7 describes firms' strategies.

Proposition 7. *Given* (6) *and* $\sigma_E < \sigma_I$, *firms face unique cutoff strategies such that:*

- *A lowest quality group of firms which are inactive in the market,* $\alpha_j \leq \sigma_E$*;*
- *A second lowest quality group of firms which are active only on the entrant,* $\alpha_j \in (\sigma_E, \underline{\alpha}]$, $\underline{\alpha} > \sigma_I$*;*
- *A second highest quality group of firms which are active only on the incumbent,* $\alpha_j \in (\underline{\alpha}, \tilde{\alpha}]$, $\tilde{\alpha} > \sigma_I$*;*
- *A highest quality group of firms which are active on both platforms,* $\alpha_j > \tilde{\alpha}$, $\tilde{\alpha} > \sigma_I$,

where $\tilde{\alpha}$ *is the minimum firm quality such that the firm makes sufficient profit to cover the entry cost. Note it is possible that* $\underline{\alpha} = \tilde{\alpha}$.

The first group of firms arises as a direct consequence of Lemma 1. Some firms never find it profitable to join any platform because they always receive zero demand from the platforms.

The second group of firms are only active on the entrant. This includes the group of firms which can never make a profit on the incumbent, $\alpha_j \in (\sigma_E, \sigma_I]$. However, it also includes some firms which despite being able to make positive profit on the incumbent prefer to join the entrant as they become a 'big fish in a small pond', rather than a 'small fish in a big pond', allowing them to capture a larger share of the surplus on the entrant than on the incumbent as they face weaker competition.

The rest of the firms are active on the incumbent as, despite setting lower unit prices, the quality of firms on the incumbent is higher and this attracts more consumers, hence allowing these firms to make larger profits on the incumbent. Finally, note that only the highest quality firms, $\alpha_j > \tilde{\alpha}$, make enough profit from joining a second platform and multi-home.

Contrasting Proposition 6 and Proposition 7 yields an interesting observation: Costly firm entry would cause firms who would otherwise multi-home to either single-home on the entrant if they were lower quality, or single-home on the incumbent if they were higher-quality. If one could observe an exogenous increase in firms entry cost (e.g. if a platform raises its prices), observing which market multi-homing firms choose to exit could provide a different method of distinguishing firm's appeal to consumers.

Implications Fix some (σ_I, σ_E) pair and suppose $\tilde{\alpha} > \underline{\alpha} > \sigma_I$. Then it must be that some firms with quality $\alpha_j \in (\sigma_I, \underline{\alpha}]$ exit the incumbent and join only the entrant. This raises the expected quality of firms on the incumbent relative to costless entry. Moreover, there exists some firms with quality $\alpha_j \in (\underline{\alpha}, \tilde{\alpha}]$ which exit the entrant and only join the incumbent. This lowers the expected quality of firms on the entrant. This means, unlike the sequential equilibrium with costless firm entry, for any (σ_I, σ_E) pair, consumers' expected consumption utility from the incumbent is higher with costly firm entry while that on the entrant is lower. Therefore, costly firm entry can limit the competitiveness of entrant platforms.

5.3 Asymmetric consumer cost

It is likely that consumers face a lower cost of joining an established platform—possibly due to switching costs, familiarity, or other peer-based network effects. Following from the model of competition where consumers single-home and platforms compete simultaneously, one can model this situation as the distribution of the cost of joining the entrant being higher than the incumbent in a first-order stochastic dominant sense. I show that the entrant prefers to develop a more precise recommender system. While this result may be difficult to observe in a dominated space like e-commerce, it is more easily observed on social media platforms. For example, TikTok has better recommendation algorithms than Instagram (Gerbaudo, 2024).

For details, see Appendix B.

6 Discussion

Environment	Monopoly		Competition	
Consumer behavior	Non-naive $\sigma^m = 1/4$	Naive $\sigma^N < \sigma^m$	Single-homing $\sigma^s > \sigma^m$	Multi-homing $\sigma^h < \sigma^m$
Exogenous shifts	Marginal costs (e) $\frac{\partial \sigma}{\partial e} > 0$		Entrant recommender system (σ_E) $\frac{\partial \sigma_I}{\partial \sigma_E} > 0$. If $\sigma_E = 0$, $\sigma_I < \sigma^m$.	

Table 1: Summary of results across competitive environments. In the monopoly environment, displaying results for naive consumers and an increase in firms' marginal cost. In the competitive environment, displaying results for consumer multi-homing behavior and exogenous selection of entrant recommender system.

Table 1 summarizes the key results of this paper. Together, these results broadly align with how recommender systems on platforms such as eBay and Amazon have evolved with market environments. When recommender systems were first introduced, consumers may not have understood their implications. Hence, early monopolist platforms, such as eBay, adopted recommender systems that purely reflected value-for-money (σ^N). As consumers learned that recommender systems did indeed reflect value-for-money and began to trust these recommendations, platforms became incentivized to design increasingly precise recommender systems (until σ^m). While early competition may not have stimulated improvements to recommender systems, as competitors became more sophisticated so did recommender systems ($\frac{\partial \sigma_I}{\partial \sigma_E} > 0$, $\sigma^s > \sigma^m$). However, if a platform is able to establish market dominance (e.g. a monopoly), recommender systems become less precise. Therefore, the lack of competition between platforms may offer an explanation for the degradation of platforms, in the form of poorer recommendations and search results, recently highlighted by the popular media.

Although entrenched monopolies may cause recommender systems to deteriorate, consumers are still weakly better off than without these systems (Corollary 2). Moreover, the lack of competition is likely only one of several factors contributing to platform degradation. For example, this paper shows that platform degradation can occur for a variety of other reasons. For example, if firms face lower marginal costs, which can arise if firms partake in cost-cutting measures, this leads to lower prices. Platforms then have an incentive to weaken competition between firms as their per-transaction revenue becomes more elastic with respect to the precision of their recommender systems. Alternatively, if consumers multi-home due to increased sophistication in comparing across websites or decreased platform loyalty, then consumers' decision to buy on a platform becomes less elastic with respect to the precision of the recommender system. This way the platform prefers less precise recommender systems to abate price competition between firms—raising prices on the platform, and its per-transaction revenue. Likewise, if recommender systems are overly complicated such that consumers cannot understand their implications, consumers' decision to join the platform becomes less elastic with respect to recommender system precision.

Together, these results suggests that policies intending on reversing platform degradation cannot simply promote competition between platforms. Instead, a number of other factors can lead to poorer recommender systems and user-experiences, and understanding the root cause of why platforms are becoming less attractive to users is necessary for a targeted policy response, if at all.

7 Related literature

This paper explores how platforms compete in the development of their recommender systems. In an emerging literature on the development of *recommender systems*, Peitz and Sobolev (Forthcoming) and Li et al. (2020) model horizontally differentiated products, Peitz and Sobolev (Forthcoming) show a platform may choose to bias its recommendation when consumers have strong preferences, and Li et al. (2020) show that a recommender system which directs consumers to the product offering the highest net utility can make some sellers worse off and lead to lower prices. Zhou and Zou (2023) studies a model of vertical differentiation and describes how the accuracy of quality-based recommendations can abate price competition by firms, leading to higher prices, making consumers worse off. Like Zhou and Zou (2023), I consider a model of vertical differentiation but consider how platforms may choose the accuracy of value-for-money based recommendations. Given this distinction, unlike them, in the monopoly setting I find more precise recommender systems lead to lower prices, making some sellers worse off which has nuanced implications for firms participation decisions. Whereas the aforementioned papers focus on a monopolist platform, to the best of my knowledge, this paper is the first to provide a tractable model of how competing platforms develop their recommender systems.

My model is perhaps most similar to Casner and Teh (Forthcoming) in that both papers similarly adopt contest success functions with some precision to model platform recommender systems. Their paper focuses its attention on how firms design their products in response to a platform's fee structure and implementation of the recommender system (or lack thereof). In contrast, my paper fixes the platform's fee structure and asks what the optimal recommender system specification is under this technology.[18]

In many settings, platforms adopt *non-price strategies*. On monopolist platforms, Johnen and Ng (2024) model value-for-money ratings and discuss how platforms can facilitate consumer ratings, and find that consumer surplus is maximized with a moderate level of facilitation. I adopt a similar notion of value-for-money but adapt it to a model of recommendations. Models of recommendations are similar to models of certification such as Celik and Strausz (2025) which discusses the optimal information disclosure by a platform when there is uncertainty over firms; and Bedre-Defolie et al. (2024) which finds certification can lead sellers to exert more effort. In the monopoly platform model, my results mirror theirs in that the platform prefers a recommender system which is somewhat precise about consumption utility, and this leads to firms setting lower prices. Nocke and Strausz (2023) characterizes when a platform is able to build its collective reputation even as individual firms have their own incentives, and this is similar to how consumers in my model are attracted to platforms which provide more precise recommendations raising the collective reputation of the platform. Hagiu and Wright (2024) looks at a platform's decision to raise consumer awareness of the presence of other sellers. Sellers then compete in prices for aware buyers. My model similarly studies how active firms to compete in prices for consumers attention, and additionally allows for the platform to make some (higher value-for-money) sellers prominent.

Whereas the aforementioned papers focus on understanding the implications of a monopolist platform adopting non-price strategies, this paper examines the role of non-price strategies by *competing two-sided platforms*. When two-sided platforms compete, Chellappa and Mukherjee (2021) and Hałaburda and Yehezkel (2013) show, when information is asymmetric, platforms prefer to hide information where possible. Considering consumers with a bias for a particular platform, Hałaburda and Yehezkel (2016) discusses the effects this can have on a platform's fee and fee structure. Choi (2010) describes how tying products can improve consumer surplus when consumers can multi-home. Many others have also explored the intricate effects of platforms deciding fee structure and setting fees in competing two-sided markets (Rochet and Tirole, 2003; Caillaud and Jullien, 2003; Armstrong, 2006; Damiano and Hao, 2008; Jullien, 2011; Amaldoss et al., 2024; Belleflamme and Peitz, 2019; Bakos and Halaburda, 2020; Belleflamme and Peitz, 2010; Teh, 2022; Karle et al., 2020; Bar-Isaac and Shelegia, Forthcoming; Choi and Jeon, 2023). This paper focuses on the specification of recommender systems.

A substantial literature starting with Armstrong et al. (2009) studies prominence in *consumer search on platform markets*. Generally, these models focus on a single prominent firm and how steering and vertical integration can influence firms' desire to become prominent. I abstract away from the search element of such models and study how platforms may make a group of firms more prominent by introducing (or reducing) noise in the market. I use this model to examine the issues of firm participation on platforms and competition between platforms. Perhaps most similar in spirit within this literature is De Corniere (2016) which considers a search model where firms pay for prominence and determine the broadness of keywords used in

[18] In Appendix B, I provide conditions for when my results hold for more general contest success functions.

consumer search. He shows there exists an equilibrium where firms prefer the same broadness of search regardless of search engine competition. If, instead, the search engine determines the broadness of keywords, it prefers less accurate keyword matching than firms. Analogously, my model explores the degree of recommendation precision on a platform but focuses on the platform's optimal precision rather than a firm's. In my mechanism, a platform directly chooses the level of precision rather than indirectly as a result of its fee structure.

More recently, there has been growing interest in the role of *algorithms*. A focus of this literature is algorithmic pricing, showing algorithms lead to tacit collusion, for which Assad et al. (2021) provides a comprehensive survey. On understanding the role of data sharing and its impact on algorithms, Bergemann and Bonatti (2024) discusses how data is necessary to drive algorithms, showing privacy rules can protect consumers from targeted advertising, and Petropoulos et al. (2023) shows information sharing between platforms can improve competition. While I do not explicitly model the role of data, in the event where the absence of data sharing can inhibit entrant platforms' ability to develop precise recommender systems, in line with Petropoulos et al. (2023), my results suggest that entrant platforms may find it difficult to attract high quality firms. However, I show entrant platforms can still create competitive pressures on the incumbent platform to improve its recommender system.

8 Conclusion

Recommender systems are an integral component of the online economy, enabling users to navigate and manage an overwhelming volume of information. I show that platforms face a trade-off when developing these systems: Recommender systems that are more precise about consumption utility intensify price competition between firms on the platform since providing more consumption utility increases demand for the firm. Lower quality firms are unable to compete and may exit the market. While both effects improve consumer surplus, which attracts more consumers to the platform, a platform charging ad valorem fees obtains lower per-transaction revenue.

On a monopoly platform, I show the platform prefers recommender systems that are more precise about consumption utility than purely value-for-money recommendations. I study how consumer naïveté can affect the specification of recommender systems. Because consumers do not fully anticipate how the recommender system affects their surplus, their decision to join the platform becomes less sensitive to how the system is specified. Hence, the platform has less incentive to develop precise systems to attract consumers, instead focusing on enabling firms to set higher prices. I also examine the role of costly production, and find this causes firms to set higher prices. Hence, softening the platform's incentive to raise prices by specifying less precise recommender systems, and instead focus on attracting consumers with more precise recommender systems.

A key contribution of this paper is its analysis of platform competition. When platforms compete, if the entrant has a recommender system that is uninformative of value-for-money, the incumbent prefers less precise systems than a monopolist does. However, as the entrant develops more precise recommender systems, the incumbent also improves its recommender system. I find a unique symmetric equilibrium in which both platforms prefer recommender systems that are more precise about consumption utility than a monopolist does, showing that competition indeed results in better recommendations. Using this insight, I discuss how modern data-driven recommender systems may prevent entrants from developing precise recommender systems, and how data portability and data-sharing obligations can result in recommender systems that are more useful for consumers. Combining my findings on monopoly and competing platforms provides insight on how market power can be a contributing factor to platform degradation, at least as it relates to how consumers may perceive recommender systems to be less useful.

I also examine the scope of consumer and firm multi-homing. First, I allow consumers to multi-home and find a unique symmetric equilibrium in which platforms prefer less precise recommendations than when consumers cannot multi-home (σ^s). This is because when consumers are able to search across platforms, each platform finds it more difficult to retain consumers and instead focuses on raising per-transaction revenues. Second, I consider firms' costly entry onto platforms. This makes multi-homing only possible for the highest quality firms, which serves to raise the expected consumption utility from the incumbent as fewer relatively lower-quality (middle and lowest) firms are active on the incumbent.

This paper focuses on how platforms strategically decide on their recommender system precision when such systems are unbiased. The paper does not explore other important regulatory issues such as platform self-preferencing or sponsored and advertised products. How platforms strategically use their recommender systems when such biases are present would be one direction for further research.

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A Proofs

Proof of Lemma 1. To prove the first statement, consider the profit function of a firm,

$$\pi(\alpha_j) = n\frac{\alpha_j - \sigma}{\int_{h\in\mathbf{N}} \alpha_h - \sigma \; d\alpha_h}(1-r)\frac{\alpha_j - \sigma}{2}.$$

Higher quality firms are both able to set higher prices and obtain a larger share of demand.

The second statement follows from λ. For some $\sigma > 0$, if $\sigma \geq \alpha_j$, firms with quality α_j are never recommended to consumers, and are inactive on the platform. Hence, only firms with $\alpha_j > \sigma$ are active on the incumbent.

To show the third statement, see that the second statement implies any $\sigma \geq 1$ means all firms receive no demand (or profits) and are inactive on the platform. □

Proof of Corollary 1. For a firm to be active, it has to make positive profit on the platform. Since firms have no outside option, they are active on the platform as long as $\pi(\lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma), p_j) \geq 0$ and $\alpha_j - p_j - \sigma \geq 0$. Since profits are increasing in α_j, there is a lowest quality firm, $\bar{\alpha}$, such that $\pi(\bar{\alpha}) = 0$, which is the highest quality firm that is inactive on the platform. All firms with quality above $\bar{\alpha}$ make strictly more profit than $\bar{\alpha}$, hence make positive profit and are active on the platform. Other firms with quality at or below $\bar{\alpha}$ do not make positive profit and are inactive. Hence, to find this cutoff quality level $\bar{\alpha}$, $\pi(\lambda(\bar{\alpha}, p_j^*, \mathbf{p}_{\text{-}j}, \sigma), p_j^*) = 0$,

$$\frac{1 + \bar{\alpha} + \bar{\alpha}^2 - 3\sigma^2}{3(1 + \bar{\alpha} - 2\sigma)}\frac{2(\bar{\alpha} - \sigma)}{(1 - \bar{\alpha})(1 + \bar{\alpha} - 2\sigma)}(1-r)\frac{\bar{\alpha} - \sigma}{2} = 0.$$

Any $\bar{\alpha}$ that makes $\pi(\lambda(\bar{\alpha}, p_j^*, \mathbf{p}_{\text{-}j}, \sigma), p_j^*) = 0$, must satisfy either $1+\bar{\alpha}+\bar{\alpha}^2-3\sigma^2 = 0$ or $\bar{\alpha}-\sigma = 0$. Since $1 + \bar{\alpha} + \bar{\alpha}^2 - 3\sigma^2 = 0$ implies the platform receives zero demand, and this would imply an inactive market, this implies all firms are inactive, a contradiction to the cutoff rule. Therefore, $\bar{\alpha} = \sigma$. □

Proof of Proposition 1. To see this, observe that $n = \frac{1+2\sigma}{3}$, $\frac{\partial n}{\partial \sigma} = \frac{2}{3} > 0$; and $\int_{\bar{\alpha}}^1 \lambda(\alpha_h, p_h^*, \mathbf{p}_{\text{-}h}, \sigma)p_h^* \; d\alpha_h = \frac{1-\sigma}{3}$, $\frac{\partial}{\partial\sigma}\int_{\bar{\alpha}}^1 \lambda(\alpha_h, p_h^*, \mathbf{p}_{\text{-}h}, \sigma)p^*(\alpha_h) \; d\alpha_h = -\frac{1}{3} < 0$. The platform's profit is $\Pi = r\frac{(1+2\sigma)(1-\sigma)}{9}$ and balancing the mass of consumers and per-transaction revenue, $\frac{\partial \Pi}{\partial \sigma} = r\frac{1-4\sigma}{9} > 0$ at $\sigma = 0$ and $\frac{\partial^2 \Pi}{\partial \sigma^2} = -\frac{4r}{9} < 0$, it sets the optimal $\sigma^M = \frac{1}{4}$. □

Proof of Corollary 2. Observe that the expected consumption utility is given by (1), $\frac{1+2\sigma}{3}$. Since only $\frac{1+2\sigma}{3}$ mass of consumers obtain this consumption utility, in expectation the total consumer surplus is $\int_0^{\frac{1+2\sigma}{3}} E[u] - c_i \; dc_i = \frac{(1+2\sigma)^2}{18}$, which is strictly increasing in σ. In equilibrium, the total consumer surplus with precise recommendations is $\frac{1}{8}$. □

Proof of Corollary 3. To see how a firm's profit changes with σ,

$$\frac{\partial \pi(\alpha_j)}{\partial \sigma} = \frac{2(1-r)(\alpha_j - \sigma)}{3(1-\sigma)^3}((\alpha_j - \sigma)(2+\sigma) - (1+2\sigma)(1-\sigma))$$

and $\frac{\partial \pi(\alpha_j)}{\partial \sigma} > 0 \Leftrightarrow \alpha_j > \hat{\alpha} \equiv \frac{(1+2\sigma)(1-\sigma)}{2+\sigma} + \sigma > \sigma$, and $\frac{\partial \pi(\alpha_j)}{\partial \sigma} = 0 \Leftrightarrow \alpha_j = \hat{\alpha}$.

To see that the cutoff $\hat{\alpha}$ is increasing in σ, $\frac{\partial \hat{\alpha}}{\partial \sigma} = \frac{5-4\sigma-\sigma^2}{(2+\sigma)^2} > 0$. $\square$

Proof of Corollary 4. From Corollary 1, only firms with quality $\alpha_j > \sigma_I$ may be active on the incumbent. Notice that although firms are able to multi-home, firms with quality $\alpha_j > \sigma_I$ do not join the entrant in addition to the incumbent. This is because for all firms, $\pi_E(\alpha_j) = 0$. Hence, if some small but positive mass of firms multi-home and join the entrant this induces more consumers to join the entrant, which draws demand away from the incumbent, reducing their own profits.

For firms with $\alpha_j \leq \sigma_I$, they are unable to be active on the incumbent. Although they make zero profit on the entrant, firms prefer selling to not and are therefore active only on the entrant. $\square$

Proof of Proposition 2. A firm's profit function becomes $\pi(\alpha_j) = n\lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma)(p_j - e)$, and the profit maximizing price is $\frac{\alpha_j - \sigma + e}{2}$. Only firms with quality $\alpha_j > \sigma + e$ are profitable, and hence active, on the platform. Consumption utility, (1), and platform profits, (3), become $E[u] = \frac{1+2\sigma-e}{3}$ and $\Pi = \frac{1+2\sigma-e}{3} r \frac{1-\sigma+2e}{3}$. The platform's optimal recommender system specification is $\sigma^* = \frac{1+5e}{4}$. $\square$

Proof of Proposition 3. To see this, observe that $\Pi_I^R = (\frac{1+2\sigma_I}{3} - \frac{\sigma_I^2}{8}) r \frac{1-\sigma_I}{3}$, $\frac{\partial \Pi_I^R}{\partial \sigma_I} = \frac{r(8-38\sigma_I+9\sigma_I^2)}{72}$, > 0 when $\sigma_I = 0$ and $\frac{\partial^2 \Pi_I^R}{\partial \sigma_I^2} = \frac{r(9\sigma_I - 19)}{36}$, < 0 for all $\sigma_I \in [0, 1]$. Therefore, there is only one feasible solution and $\sigma_I^R = \frac{2}{9}$. $\square$

Proof of Corollary 5. A consumer joining the incumbent expects a transaction of value $\frac{1+2\sigma_I}{3}$, and those joining the entrant expect a transaction value of $\frac{\sigma_I}{2}$. Note that the joint density of entry costs from both platforms is 1. Taking into consideration the consumers' choice of platform, the total consumer surplus from the incumbent is

$$\int_0^{\frac{\sigma_I}{2}} \int_0^{\frac{2+\sigma_I}{6}+c_E} \frac{1+2\sigma_I}{3} - c_I \, dc_I \, dc_E + \int_{\frac{\sigma_I}{2}}^1 \int_0^{\frac{1+2\sigma_I}{3}} \frac{1+2\sigma_I}{3} - c_I \, dc_I \, dc_E = \frac{8+32\sigma_I+32\sigma_I^2-3\sigma_I^3}{144}.$$

and the total consumer surplus from the entrant is

$$\int_{\frac{1+2\sigma_I}{3}}^{\frac{2+\sigma_I}{6}} \int_0^{\frac{\sigma_I}{2}-c_I} \frac{\sigma_I}{2} - c_E \, dc_E \, dc_I + \int_{\frac{1+2\sigma_I}{3}}^1 \int_0^{\frac{\sigma_I}{2}} \frac{\sigma_I}{2} - c_E \, dc_E \, dc_I = \frac{\sigma_I(2+11\sigma_I-7\sigma_I^2)}{72}.$$

Therefore, in equilibrium, consumer surplus on the incumbent is $\frac{253}{2187}$, consumer surplus on the entrant is $\frac{83}{6561}$, and the total consumer surplus is $\frac{842}{6561}$. $\square$

Proof of Proposition 4. I show that there exists a unique symmetric equilibrium. To see this, I first show that the equilibrium cannot be asymmetric. I then show the symmetric equilibrium.

Without loss of generality, suppose to a contradiction that there is an asymmetric equilibrium such that $\sigma_I > \sigma_E$. This implies $E[u_I] = \frac{1+2\sigma_I}{3} > \frac{1+2\sigma_E}{3} = E[u_E]$, and the mass of consumers joining either platform is $n_I = \frac{5-4\sigma_E-4\sigma_E^2+12\sigma_I}{18}$ and $n_E = \frac{(1+2\sigma_E)(5+2\sigma_E-4\sigma_I)}{18}$. The profit function of either platform becomes $\Pi_I = n_I r \frac{1-\sigma_I}{3}$ and $\Pi_E = n_E r \frac{1-\sigma_E}{3}$, which are concave in σ_I and σ_E respectively. The best response function of either platform is $\sigma_I = \frac{7+4\sigma_E+4\sigma_E^2}{24}$ and

$\sigma_E = \frac{4\sigma_I+\sqrt{37-44\sigma_I+16\sigma_I^2}-4}{6}$, since all other solutions lie outside the range of $\sigma_k \in [0,1]\ \forall k$. This solves $\sigma_I = \frac{5-3\sqrt{2}}{2}$ and $\sigma_E = \frac{5-3\sqrt{2}}{2}$. A contradiction.

Suppose instead the equilibrium is symmetric, such that $E[u_I] = \frac{1+2\sigma_I}{3}$, and $E[u_E] = \frac{1+2\sigma_E}{3}$, then the mass of consumers joining either platform is $n_k = \frac{5+12\sigma_k-4\sigma_{\text{-}k}-4\sigma_{\text{-}k}^2}{18}$. The profit function of either platform becomes $\Pi_k = n_k r \frac{1-\sigma_k}{3}\ \forall k$, which is concave in σ_k. The best response function of platform k is $\sigma_k = \frac{7+4\sigma_{\text{-}k}+4\sigma_{\text{-}k}^2}{24}$. Then $\sigma_k = \frac{5-3\sqrt{2}}{2}$ is the only solution which satisfies the condition $\sigma_k \in (0,1)$. Note that $\Pi_k^s = r(\frac{3}{2}-\sqrt{2})$. □

Proof of Proposition 5. First note that consumers always receive a positive consumption utility from purchasing the product which is recommended by the platform. This means consumers always purchase in the last stage. The expected consumption utility from joining either platform is given by (1). Given this, I search for a symmetric equilibrium in platform strategies where $\sigma_I = \sigma_E = \sigma^h$. To save on notation, I drop the platform subscripts. Recall from Lemma 1 that all firms with quality above σ^h will join both platforms. In any such symmetric equilibrium, it must be that the expected consumption utility from either platform is the same, $E[u^h]$. Therefore, the initial mass of consumers joining either platform is $n_{int} = E[u^h](1-\frac{E[u^h]}{2})$. Note this means consumers join the platform for which they have a lower cost of joining first.

Upon joining the platform k, some firm with quality α' is recommended to the consumer with probability $\lambda(\alpha', p_k, \mathbf{p}_k, \sigma_k)$. Consumption utility from this interaction is $\alpha' - p_k$. Then the expected gains from search is $E[\Delta(\alpha')] = \int_{\alpha'}^1 (\alpha_h - p_h)\lambda(\alpha_h, p_h, \mathbf{p}_{\text{-}h}, \sigma_{\text{-}k})\ d\alpha_h$. The consumer joins the second platform if $E[\Delta(\alpha')] > c_{i,\text{-}k}$. Hence, the probability a consumer searches is $E[\Delta(\alpha')]$. The mass of consumers who stop and purchase from firm α' on platform k is $n_{immediate}(\alpha', p, \mathbf{p}) = n_{int}\lambda(\alpha', p, \mathbf{p})(1-E[\Delta(\alpha')])$.

Consumers who continue searching return to firm α' on platform k if the recommended product on platform -k has quality below α'. This occurs with probability $\Lambda(\alpha', p, \mathbf{p})$, where $\Lambda(\alpha', p, \mathbf{p}) = \int_{\sigma^h}^{\alpha'} \lambda(\alpha_h, p_h, \mathbf{p}_{\text{-}h})\ d\alpha_h$. Hence, the return demand is $n_{return}(\alpha', p, \mathbf{p}) = n_{int}\lambda(\alpha', p, \mathbf{p})E[\Delta(\alpha')]\Lambda(\alpha', p, \mathbf{p})$.

Finally, consider the new demand from platform -k. Since the platforms are symmetric, the mass of consumers initially joining the platforms is the same. Suppose a consumer is recommended some α'' on platform -k. Then their expected gains from search are $E[\Delta(\alpha'')]$ and the mass of consumers searching k after visiting $-k$ is $\int_{\sigma^h}^1 n_{int}\lambda(\alpha_h, p_h, \mathbf{p}_{\text{-}h})E[\Delta(\alpha_h)]\ d\alpha_h$. On platform k, these consumers are matched with firm α' with probability $\lambda(\alpha', p, \mathbf{p})$. They choose to purchase from α' if $\Pr(\alpha' \geq \alpha'') = \Lambda(\alpha', p, \mathbf{p})$. Therefore, the mass of new consumers purchasing from α' on platform k is $n_{new}(\alpha', p, \mathbf{p}) = \int_{\sigma^h}^1 n_{int}\lambda(\alpha_h, p_h, \mathbf{p}_{\text{-}h})E[\Delta(\alpha_h)]\ d\alpha_h \times \lambda(\alpha', p, \mathbf{p})\Lambda(\alpha', p, \mathbf{p})$.

Therefore the total mass of consumers engaging with firm α' on platform k is $n(\alpha', p, \mathbf{p}) = n_{immediate} + n_{return} + n_{new} =$

$$n_{int}\lambda(\alpha', p, \mathbf{p})\left(1 - E[\Delta(\alpha')](1-\Lambda(\alpha', p, \mathbf{p})) + \Lambda(\alpha', p, \mathbf{p})\int_{\sigma^h}^1 \lambda(\alpha_h, p_h, \mathbf{p}_{\text{-}h})E[\Delta(\alpha_h)]\ d\alpha_h\right).$$

Consider the firm α' pricing strategy, observe that its profit function is $n(\alpha', p, \mathbf{p})p(1-r)$. Because firms are only able to unilaterally influence $\lambda(\alpha', p, \mathbf{p})$ through its numerator, a firm's optimal pricing strategy is the same as in the main model, $p^*_{j,k} = \frac{\alpha_j - \sigma_k}{2}$.

Finally, I turn to the platform's problem. In the symmetric equilibrium, platforms select $\sigma_k = \sigma^h$ to maximize $\Pi = r \int_{\sigma^h}^1 n(\alpha_h, p_h, \mathbf{p}_{\text{-}h}) p(\alpha_h)\ d\alpha_h$. Then applying the functional form of λ and accounting for prices,

$$\begin{aligned}
\lambda(\alpha_j) &= \frac{2(\alpha_j - \sigma^h)}{(1-\sigma^h)^2} \\
\Lambda(\alpha_j) &= \frac{(\alpha_j - \sigma^h)^2}{(1-\sigma^h)^2} \\
E[\Delta(\alpha_j)] &= \frac{(1-\alpha_j)(1+\alpha_j+\alpha_j^2 - 3\sigma^{h2})}{3(1-\sigma^h)^2} \\
n_{int} &= \frac{1+2\sigma^h}{3}(1 - \frac{1+2\sigma^h}{6}).
\end{aligned}$$

Therefore, the platform's problem evaluates to become $\frac{(1-\sigma^h)(5-2\sigma^h)(1+2\sigma^h)(4319+181\sigma^h)}{226800}$, and $\sigma^h = 0.144$. which is a local maxima with the relevant range of $\sigma \in [0,1)$. □

Proof of Proposition 6. Following backward induction, note that consumers join the platform which give them the highest expected consumption utility less cost of joining and always purchase from the recommended firm.

I now show that all firms with quality $\alpha_j > \sigma_k$ are active on platform k. On the platform k, any firm with quality larger than σ_k. Since firms face no cost of multi-homing and on the margin firms are unable to influence demand for a platform, the benefit from joining an additional platform is larger than the cost of doing so. Therefore, any firm with quality $\alpha_j > \sigma_k$ is active on the platform k.

Next, consider the strategies of the incumbent. Here consider two cases, $E[u_I] \geq E[u_E]$ and $E[u_E] > E[u_I]$. Since all firms join a platform if they are able to, $E[u_k] = \frac{1+2\sigma_k}{3}$, and the mass of consumers joining either platform is

$$n_k = \begin{cases} \frac{5+12\sigma_k - 4\sigma_{\text{-}k} - 4\sigma_{\text{-}k}^2}{18} & \text{if } \sigma_k \geq \sigma_{\text{-}k} \\ \frac{(1+2\sigma_k)(5+2\sigma_k - 4\sigma_{\text{-}k})}{18} & \text{otherwise} \end{cases}$$

and the profit of the incumbent is

$$\Pi_I = \begin{cases} r\frac{(1-\sigma_I)(5+12\sigma_I - 4\sigma_E - 4\sigma_E^2)}{54} & \text{if } \sigma_I \geq \sigma_E \\ r\frac{(1-\sigma_I)(1+2\sigma_I)(5+2\sigma_I - 4\sigma_E)}{54} & \text{otherwise.} \end{cases}$$

Then

$$\sigma_I = \begin{cases} \frac{7+4\sigma_E+4\sigma_E^2}{24} & \text{if } \sigma_I \geq \sigma_E \\ \frac{4\sigma_E + \sqrt{37-44\sigma_E+16\sigma_E^2} - 4}{6} & \text{otherwise} \end{cases}$$

solves the incumbent's problem.

Turning to the first stage of the game, the entrant maximizes its profit

$$\Pi_E = \begin{cases} r\frac{(1+2\sigma_E)(1-\sigma_E)(23+8\sigma_E-4\sigma_E^2)}{324} & \text{if } \sigma_I \geq \sigma_E \\ r\frac{(1-\sigma_E)(8-16\sigma_E^2+\sqrt{37-44\sigma_E+16\sigma_E^2}+4\sigma_E(20-\sqrt{37-44\sigma_E+16\sigma_E^2}))}{243} & \text{otherwise.} \end{cases}$$

Then in equilibrium, σ_E and σ_I are

$$\sigma_E = \begin{cases} 0.3113 & \text{if } \sigma_I \geq \sigma_E \\ 0.3583 & \text{otherwise,} \end{cases}$$

$$\sigma_I = \begin{cases} 0.3597 & \text{if } \sigma_I \geq \sigma_E \\ 0.3765 & \text{otherwise.} \end{cases}$$

Note that if $\sigma_E > \sigma_I$, then there is a contradiction. Therefore, in equilibrium $\sigma_E = 0.311$ and $\sigma_I = 0.356$, which satisfy $\sigma_k \in (0, 1)$. The profits of the platforms are $\Pi_E = 0.0866r$ and $\Pi_I = 0.0911r$.

$\square$

Proof of Proposition 7. I search for an equilibrium in firm cutoff strategies. Recall that consumers join the platform which provides them with the highest expected consumption utility less cost of joining the platform, and purchase from the recommended firm.

Consider now the firms' problem. First observe that firms are unable to unilaterally affect demand with their prices. Therefore, a firm on either platform sets the price $\frac{\alpha_j-\sigma_k}{2}$. Now note that a firm either joins no, one, or both platforms. Recall from Lemma 1 that on a given platform higher quality firms obtain higher profits. This means that only firms with sufficiently high quality are able to make enough profits to join both platforms. Let the cutoff of firms that multi-home be $\tilde{\alpha}$ such that all firms with quality above $\tilde{\alpha}$ prefer to multi-home. Also from Lemma 1, firms with quality below σ_k are inactive on platform k therefore it must be that for any $\alpha_j \leq \min\{\sigma_I, \sigma_E\}$, these firms are inactive, and for any firm with quality $\alpha_j \in (\sigma_{\text{-}k}, \sigma_k]$ where $\sigma_{\text{-}k} < \sigma_k$ these firms are only active on the platform -k.

It remains to show which platform the firms with quality $\alpha_j \in (\sigma_k, \tilde{\alpha}]$ choose to join. When choosing which platform the join, the firms evaluate the following: $n_k(1-r)\frac{\alpha_j-\sigma_k}{\int_{h\in\mathbf{N}_k}\alpha_h-\sigma_k\,d\alpha_h}\frac{\alpha_j-\sigma_k}{2}$ selecting the platform which provides it with the highest profit. Without loss of generality, suppose that $\sigma_k > \sigma_{\text{-}k}$ and suppose there is a cutoff firm $\underline{\alpha} \geq \sigma_k$ such that all firms above join the incumbent and those below do not. Since the profits of the incumbent is increasing in α, such firm must weakly prefer the incumbent,

$$\begin{aligned} n_k\frac{\underline{\alpha}-\sigma_k}{\int_{\underline{\alpha}}^1 \alpha_h-\sigma_k\,d\alpha_h}\frac{\underline{\alpha}-\sigma_k}{2} &\geq n_{\text{-}k}\frac{\underline{\alpha}-\sigma_{\text{-}k}}{\int_{\tilde{\alpha}}^1 \alpha_h-\sigma_{\text{-}k}\,d\alpha_h+\int_{\sigma_{\text{-}k}}^{\underline{\alpha}}\alpha_h-\sigma_{\text{-}k}\,d\alpha_h}\frac{\underline{\alpha}-\sigma_{\text{-}k}}{2} \\ \Leftrightarrow n_k\frac{(\underline{\alpha}-\sigma_k)^2}{\int_{\underline{\alpha}}^1 \alpha_h-\sigma_k\,d\alpha_h} &\geq n_{\text{-}k}\frac{(\underline{\alpha}-\sigma_{\text{-}k})^2}{\int_{\tilde{\alpha}}^1 \alpha_h-\sigma_{\text{-}k}\,d\alpha_h+\int_{\sigma_{\text{-}k}}^{\underline{\alpha}}\alpha_h-\sigma_{\text{-}k}\,d\alpha_h} \\ \Leftrightarrow X(\underline{\alpha}-\sigma_k)^2 &\geq (\underline{\alpha}-\sigma_{\text{-}k})^2, \end{aligned} \tag{6}$$

where $X = \frac{n_k(\int_{\tilde{\alpha}}^{1} \alpha_h - \sigma_{\text{-}k} \; d\alpha_h + \int_{\sigma_{\text{-}k}}^{\underline{\alpha}} \alpha_h - \sigma_{\text{-}k} \; d\alpha_h)}{n_{\text{-}k}(\int_{\underline{\alpha}}^{1} \alpha_h - \sigma_k \; d\alpha_h)}$.

To show that the equilibrium in cutoff strategies exists, it must be that any firm with quality $\alpha_j > \underline{\alpha}$ does not prefer to switch from the incumbent to the entrant. To see this, consider the following: suppose to a contradiction there exists firms of some quality $\alpha'_j \in (\underline{\alpha}, \tilde{\alpha})$ which prefers to join the entrant instead. Note that such firm cannot unilaterally change the mass of consumers joining the platform nor how other firms join the platform. Hence, if (6) holds, it must be that the firm only switches if

$$(\alpha'_j - \sigma_{\text{-}k})^2 > (\alpha'_j - \sigma_k)^2 X \Leftrightarrow \frac{\alpha'_j - \sigma_{\text{-}k}}{\alpha'_j - \sigma_k} > \sqrt{X}.$$

Note from (6) that $\sqrt{X} \geq \frac{\underline{\alpha} - \sigma_{\text{-}k}}{\underline{\alpha} - \sigma_k}$, then it is only possible for $\frac{\alpha'_j - \sigma_{\text{-}k}}{\alpha'_j - \sigma_k} > \frac{\underline{\alpha} - \sigma_{\text{-}k}}{\underline{\alpha} - \sigma_k} \Leftrightarrow \frac{\underline{\alpha} - \sigma_k}{\alpha'_j - \sigma_k} > \frac{\underline{\alpha} - \sigma_{\text{-}k}}{\alpha'_j - \sigma_{\text{-}k}}$ which cannot be true because $\sigma_k > \sigma_{\text{-}k}$. Therefore there is no profitable deviation for firms of α'_j quality. $\square$

B Suitable as accompanying appendix

B.1 Search based microfoundation

The paper assumes that consumers and firms interact in a probabilistically manner. One can consider a search process as the microfoundation for this interaction. Suppose consumers conduct sequential search with recall. The first search is free, and all subsequent search costs θ. When consumers search, they discover firms following the search process λ as defined in the main paper.

As is standard in the literature, when consumers face such as search process they stop searching whenever their expected gains from search is less than or equal to the cost of search. Define an R which represents the utility required to induce such a stopping rule.

Now turn our attention to a firm maximizing profits. As before, the mass of consumers entering the market is independent of a firm's pricing strategy because, in expectation, a firm is unable to unilaterally deviate in prices to affect the mass of consumers entering the market. Additionally, to ensure demand, the firm has to provide at least as much surplus as R. Therefore, the firm maximizes its profits $\pi(\alpha_j) = p_j \lambda_j n$ subject to the constraints $\alpha_j - p_j \geq R$ and $p_j > 0$. The optimal pricing strategy is $p_j^* = \max\{\min\{\frac{\alpha_j - \sigma}{2}, \alpha_j - R\}, 0\}$.

Since $p_j > 0$, firms are only active on the market if $\alpha_j > \max\{\sigma, R\}$. Then if $R > \sigma$, those with $R < \alpha_j < 2R - \sigma$ set the price $\alpha_j - R$, and those with $\alpha_j \geq 2R - \sigma$ set the price $\frac{\alpha_j - \sigma}{2}$. And if $\sigma > R$, all firms with $\alpha_j > \sigma$ are active on the platform, selling at price $p_j = \frac{\alpha_j - \sigma}{2}$.

For tractability, focus on the situation where $\sigma > R$. This means that consumers' expected utility is given by $\int_\sigma^1 \frac{\alpha_j - p_j - \sigma}{\int_\sigma^1 \alpha_h - p_h - \sigma \, d\alpha_h} (\alpha_j - p_j) \, d\alpha_j = \frac{1+2\sigma}{3}$. Then the reservation value for consumers is $R \equiv \frac{1+2\sigma}{3} - \theta$. As in the main model, the platform's profit function is $\frac{1+2\sigma}{3} r \frac{1-\sigma}{3}$ and, in equilibrium, $\sigma = \frac{1}{4}$. Then ensuring $\sigma > R$, we require that $\theta > \frac{1}{4}$ for this equilibrium to hold.

This exercise highlights how recasting this probabilistic function as the density of a search outcome can lead to very familiar results from the search literature. Namely, in equilibrium, consumer stop searching after inspecting the first product.

Therefore, the proposed contest fashion of framing the recommender function can be microfounded in a model of directed consumer search.

No recommender system When the platform does not implement a recommender system, I search for the consumer surplus maximizing equilibrium where firms follow a symmetric pricing strategy. Doing so allows me to find the smallest possible change in consumer surplus following the implementation of the recommender system. Moreover, this seems to be the most relevant benchmark as many regulators seem to focus on consumer outcomes.

Following the backward induction logic, consumers only purchase from firms with quality $\alpha > p(\alpha)$ and the mass of consumers active on the platform is $n = \int_{j \in \mathbf{N}} \frac{\alpha_j - p(\alpha_j)}{\int_{h \in \mathbf{N}} 1 \, d\alpha_h} \, d\alpha_j$. The firm j makes profit $n(1-r) \frac{p(\alpha_j)}{\int_{h \in \mathbf{N}} 1 \, d\alpha_h}$ and the profits of the platform is given by $nr \int_{j \in \mathbf{N}} \frac{p(\alpha_j)}{\int_{h \in \mathbf{N}} 1 \, d\alpha_h} \, d\alpha_j$.

Total consumer surplus is given by $\int_0^n E[u] - c \, dc$. Let $\overline{p} = \int_{j \in \mathbf{N}} \frac{p(\alpha_j)}{\int_{h \in \mathbf{N}} 1 \, d\alpha_h} \, d\alpha_j$, then consumer surplus can be rewritten as $(\int_{j \in \mathbf{N}} \frac{\alpha_j}{\int_{h \in \mathbf{N}} 1 \, d\alpha_h} \, d\alpha_j - \overline{p})^2/2$. Note that $1 > \alpha > p(\alpha)$, it must be that consumer surplus is decreasing in $\overline{p}$. To maximize consumer surplus, $p(\alpha) = 0$ such that $\overline{p}$ is minimized. In equilibrium, consumer surplus is $1/8$.

Therefore, the consumer surplus maximizing price is $p(\alpha) = 0$ such that all firms are active in the market. Firms and the platform make zero profits, and total welfare is driven by consumer surplus, 1/8.

B.2 Transparency and public education

The main analysis assumes consumers fully understand the equilibrium implications of how platforms develop recommender systems. However, given the complexity of recommender systems, there are concerns that consumers may not fully understand its implications. I model such naive consumers as consumers who may observe they are receiving useful recommendations from the platform but do not anticipate how firms adjust prices in response to the platform's recommender system. In other words, while a platform may select some $\sigma \neq 0$ and firms may price their products in response to this specification, naive consumers fail to recognize how firms account for σ in their pricing strategy, and over anticipate firm exit.

Recall that consumers join the platform based on their expected consumption utility, such that athey are unable to unilaterally affect the mass of consumers joining the platform. Hence a firm's decision to be active and its price is independent of consumers decision to join the platform. Therefore, firms adopt the same pricing strategy regardless of consumers' naïveté, $p_j^* = \frac{\alpha_j - \sigma}{2}$ and there is a unique cutoff $\bar{\alpha} = \sigma$ such that only firms with quality above the cutoff are active on the platform (Corollary 1).

Naifs failing to anticipate how σ affects prices, means they believe each firm sets $p_j^N = \frac{\alpha_j}{2}$, and expect consumption surplus of $\frac{\alpha_j}{2}$ from each transaction. They anticipate to be matched to firms following

$$\lambda(\alpha_j, p_j^N, \mathbf{p}_{-j}^N, \sigma) = \begin{cases} \frac{\alpha_j - p_j^N - \sigma}{\int_{h \in \mathbf{N}^N} \alpha_h - p_h^N - \sigma \; d\alpha_h} & \text{if } \alpha_j - p_j^N - \sigma \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, they believe firms are active only if $\alpha_j > 2\sigma$. And naifs expected consumption utility from the platform is $\int_{2\sigma}^1 \frac{\alpha_j - 2\sigma}{\int_{2\sigma}^1 \alpha_h - 2\sigma \; d\alpha_h} \frac{\alpha_j}{2} \; d\alpha_j$, the mass of naifs joining the platform is $\frac{1+\sigma}{3}$.

Corollary 6. *Naifs' decision to join a platform differ from non-naifs in two ways: (i) they expect the consumption utility of $\alpha_j - p_j^N = \frac{\alpha_j}{2}$ from firm α_j; (ii) they overestimate the amount of screening, and believe only firms with $\alpha_j > 2\sigma$ are active.*

Proof of Corollary 6. Naifs' expected consumption utility is $E[u^N] = \int_{h \in \mathbf{N}^N} \lambda^N(\alpha_h)(\alpha_h - p^N(\alpha_h)) \; d\alpha_h$. Effect (i) is immediate as they believe they receive $\alpha_j - \frac{\alpha_j}{2} = \frac{\alpha_j}{2}$ from a transaction with firms of quality α_j. To see effect (ii) consider the condition for firm exit. Firms exit if (a) they have to set negative prices, or (b) they receive no demand. Since naifs believe $p^N = \frac{\alpha_j}{2} > 0$, they do not anticipate firm exit due to negative prices. Firms receive demand based on $\lambda(\alpha_j)$, this means naifs believe firms become inactive if $\frac{\alpha_j}{2} - \sigma < 0 \Leftrightarrow \alpha_j < 2\sigma$. Therefore, $n^N = E[u^N] = \frac{1+\sigma}{3}$. □

The first effect in Corollary 6 shows how naifs, failing to account for how recommender systems more precise of value-for-money can drive price competition, anticipate firms set higher prices. Thus, conditional on interacting with a particular firm, consumers expect less value. The second effect in Corollary 6 discusses how naive consumers overestimate the degree of screening which occurs. Because firms are only recommended to consumers if $\alpha_j - p_j^N - \sigma > 0 \Leftrightarrow \alpha_j > 2\sigma$, consumers overestimate the quality of firms on the platform (recall from Corollary 1 firms join if $\alpha_j > \sigma$). This second effect dominates and, in expectation, naifs are more willing to join the platform.

Proposition 8. *There exists a unique subgame perfect Nash equilibrium where a monopolist platform facing naive consumers sets* $\sigma^N = 0$.

Proof of Proposition 8. To see this, observe that the mass of consumers joining the platform is $\frac{1+\sigma}{3}$. The platforms and firms correctly anticipate firm's strategy and all firms with quality $\alpha_j > \sigma$ join the platform. Thus the platform's profit function is $\Pi = r\int_\sigma^1 \frac{1+\sigma}{3} \frac{\alpha_j - \sigma}{\int_\sigma^1 \alpha_h - \sigma \, d\alpha_h} \frac{\alpha_j - \sigma}{2} \, d\alpha_j = r\frac{1+\sigma}{3}\frac{1-\sigma}{3}$. Then taking the first derivative with respect to σ, $\frac{-2\sigma}{9} < 0$ indicating that the platform prefers $\sigma = 0$. □

For some intuition, first notice that firms' optimal strategy is independent of consumer naïveté and the platform's per-transaction revenue is decreasing in σ. Then observe that naifs are inherently more willing to join the platform, the mass of naifs joining the platform is less elastic than non-naifs. In other words, a more precise recommender system improves demand from naive consumers by a smaller amount. This translates to a smaller benefit to platforms. Therefore, it is immediate to see that a platform prefers less precise recommender systems when it faces naive consumers.

Observe that platform's profit when consumers are naive is $\frac{r}{9}$, which is lower than when consumers are not naive, $\frac{r}{8}$. Further, recall from Corollary 2, consumer surplus improves when recommender systems are more precise, and the recommender system has to be sufficiently precise for consumers to be as well off as no recommender system. Hence, when consumers are naive, consumer surplus is 1/18. Their overconfidence in the recommender system can lead to worse consumer surplus outcomes than no recommender systems.

Share of naive users It is possible to consider that not the entire share of users are naive, and only some β fraction of users are naive. This way, only a share of users behave as in Corollary 6, and all others behave as in the main model. In this case, the platform's profit function becomes $\Pi = (\beta\frac{1+\sigma}{3} + (1-\beta)\frac{1+2\sigma}{3})r\frac{1-\sigma}{3}$, and the profit maximizing level of precision is $\sigma = \frac{1-\beta}{2(2-\beta)}$ which is decreasing in β. This shows that when users are partially naive, the optimal level of precision is between the naive level and the monopoly level, $\sigma \in (\sigma^N, \sigma^m)$, and the level of precision is decreasing in the share of naive users.

B.3 No free returns

I show the effects of consumers not having free returns by relaxing the assumption that firms must offer a positive consumption utility to get the attention of consumers. Instead, consumers have a probability of engaging with all firms, this probability is increasing in the value-for-money the firms offer and even those with negative value-for-money have a positive probability engaging with consumers. To capture this, let

$$\lambda(\alpha_j, p_j, \mathbf{p}_{-j}, \sigma) = \frac{\alpha_j - p_j - \sigma + \overline{u}}{\int_{h \in \mathbf{N}} \alpha_h - p_h - \sigma + \overline{u} \; d\alpha_h}$$

where the additional $\overline{u}$ is the highest consumption utility consumers can get from a particular firm in equilibrium. This allows all firms to have at least some positive demand. Recall then that some mass of consumers choose to join the platform and firms are unable to unilaterally influence this mass of consumers. The firms maximize their profits

$$(1-r)n \frac{\alpha_j - p_j - \sigma + \overline{u}}{\int_{h \in \mathbf{N}} \alpha_h - p_h - \sigma + \overline{u} \; d\alpha_h} p_j$$

by setting the price $\frac{\alpha - \sigma + \overline{u}}{2}$, rewriting firms profit function,

$$(1-r)n \frac{(\alpha_j - \sigma + \overline{u})^2}{2 \int_{h \in \mathbf{N}} \alpha_h - \sigma + \overline{u} \; d\alpha_h}.$$

Observe that the firms profits are increasing in α_j and note that a firm is only active if $\alpha_j - \sigma + \overline{u} > 0$. Further, consumption utility from α_j is $\alpha_j - p_j = \frac{\alpha_j + \sigma - \overline{u}}{2}$ which is increasing in α. Therefore, $\overline{u} \equiv \frac{1+\sigma}{3}$, the equilibrium consumption utility received from the highest quality firm. I now check which firms are active at a given σ. Firms which are active require $\alpha_j - \sigma + \overline{u} > 0 \Leftrightarrow \alpha_j > \frac{2\sigma - 1}{3}$. Therefore all firms are active unless $\sigma > \frac{1}{2}$.

Suppose first that $\sigma \leq \frac{1}{2}$ such that all firms are active. Then consumption utility and the mass of consumers joining the platform evaluates to $n = \frac{2+4\sigma-4\sigma^2}{15-12\sigma}$. The firm's profit function becomes $\pi(\alpha_j) = (1-r)n\frac{(1+3\alpha_j-2\sigma)^2}{3(5-4\sigma)}$. The platform's profit function is $r\frac{2+4\sigma-4\sigma^2}{15-12\sigma}\frac{7-10\sigma+4\sigma^2}{15-12\sigma}$, which is strictly increasing over $\sigma \in [0, \frac{1}{2}]$. Therefore, it can never be optimal for the platform to set such a σ.

Now suppose otherwise, that $\sigma > \frac{1}{2}$. The mass of consumers joining the platform and the profits of a firm with quality α_j are

$$n = \frac{1+4\sigma}{9}$$
$$\pi(\alpha_j) = (1-r)\frac{1+4\sigma}{9}\frac{(1+3\alpha_j-2\sigma)^2}{4(2-\sigma)^2}.$$

This means the platform's profit function is

$$r\frac{1+4\sigma}{9}\frac{2(2-\sigma)}{9}$$

and the profit maximizing level of σ is $\frac{7}{8}$. As before, I recover the standard two-sided platform arguments that the platform is balancing elasticities across both sides of the market. Importantly, in this scenario the platform also prefers only positive σ, recommender systems that are more precise of value-for-money.

B.4 Uninformative recommendations and distributional assumptions

The main model restricts $\sigma \in \mathbb{R}_+$ such that recommender systems are at least as precise as value-for-money. Relaxing this assumption, allow $\sigma \in \mathbb{R}$. Further, to understand how consumer costs affects the platform's decision, relax the assumption that $c_i \sim U[0,1]$ and allow this to follow some generic distribution, B with support $[0,1]$. Lemma 9 addresses the question of when a monopolist platform may prefer recommendations less precise than value-for-money.

Proposition 9. *A monopolist platform prefers recommendations which are more precise than value-for-money unless the distribution of consumer cost is too 'bottom-heavy'. In the special case where B is uniform, a monopolist always prefers recommendations which are more precise than value-for-money.*

The intuition is simple, suppose consumer cost is bottom-heavy, that is most consumers have low cost of joining the platform. Then a platform improving the precision of its recommender system is only able to attract marginally more consumers. However, recall more precise recommender systems lead firms to compete more aggressively in prices, which lowers the platform's per-transaction revenue. Hence, if the platform is only able to attract marginally more consumers, it may instead prefer uninformative recommender systems.

Proof of Proposition 9. I first consider the special case where B is uniform $[0,1]$. Observe from (4) that if $\sigma < 0$ consumers only purchase from a firm if $\alpha_j + \sigma > 0$. Otherwise, consumers may purchase and return the product, which is equivalent to zero demand. To ensure there is at least some firm where consumers purchase, it must be $\sigma > -1$. This also implies that only firms with $\alpha_j > -\sigma$ are active on the platform. Next observe that firms are only recommended to consumers with positive probability if $\alpha_j - \sigma > 0$. To ensure there is at least some firm which is recommended to consumers, it must be $\sigma < 1$. Turning to the firm problem, firms join the platform as long as they make positive profit. There is no direct sales channel or outside option. Firms are unable to individually influence platform demand, n. To maximize their profits, firms set $p^* = \frac{\alpha_j - \sigma}{2}$. This means firms want to be active whenever $\alpha_j > \sigma$. As a result, if $\sigma < 0$, all firms want to be active in the market, but only those with $\alpha_j > -\sigma$ make sales. Now consider a platform strategy. Note that if $\sigma \geq 0$, I recover the result in the main model. Suppose the platform sets $\sigma < 0$. Then $E[u] = \frac{1-\sigma-2\sigma^2}{3(1-3\sigma)}$. The platform's profit function is $\Pi = \frac{1-\sigma-2\sigma^2}{3(1-3\sigma)} r \frac{1-4\sigma+7\sigma^2}{3(1-3\sigma)}$. This is strictly increasing in σ, $\frac{\partial \Pi}{\partial \sigma} = \frac{1+3\sigma+3\sigma^2-59\sigma^3+84\sigma^4}{9(1-3\sigma)^3} > 0 \; \forall \sigma \in (-1,0)$. Therefore, $\sigma < 0$ cannot be an equilibrium.

Now consider the case where consumer cost follows a generic distribution B and let b represent its PDF.

Solving the game by backward induction, recall that consumers form expected consumption utility following (1) and firms set prices $\frac{\alpha_j - \sigma}{2}$, this is true for all $\sigma \in [-1,1]$. Then suppose $\sigma < 0$. Consumers expected consumption utility is $\frac{1-\sigma-2\sigma^2}{3(1-3\sigma)}$. The mass of consumers joining the platform is $B(\frac{1-\sigma-2\sigma^2}{3(1-3\sigma)})$, and since $b > 0$ and $\frac{1-\sigma-2\sigma^2}{3(1-3\sigma)}$ is increasing in σ, it must be that the mass of consumers joining the platform is increasing in σ. The platform's profit function is now $\Pi = rB(\frac{1-\sigma-2\sigma^2}{3(1-3\sigma)})\frac{1-4\sigma+7\sigma^2}{3(1-3\sigma)}$. Then $\frac{\partial \Pi}{\partial \sigma} = rb(\cdot)\frac{2(1-2\sigma+3\sigma^2)}{3(1-3\sigma)^2}\frac{1-4\sigma+7\sigma^2}{3(1-3\sigma)} - \frac{rB(\cdot)(1-14\sigma+21\sigma^2)}{3(1-3\sigma)^2}$. This is positive as long as b is sufficiently large. Then considering the case where $\sigma > 0$, we know the platform's profit function is $\Pi = rB(\frac{1+2\sigma}{3})\frac{1-\sigma}{3}$, and $\frac{\partial \Pi}{\partial \sigma} = rb(\cdot)\frac{2}{3}\frac{1-\sigma}{3} - \frac{rB(\cdot)}{3}$. Evaluating this at

$\sigma = 0$, we have $rb(\frac{1}{3})\frac{2}{9} - \frac{rB(\frac{1}{3})}{3}$, which is positive if b is sufficiently large. In other words, that the firm can attract sufficiently many consumers following an increase in σ, it is willing to make its recommender system at least as precise as value-for-money. The distribution of consumers cannot be too 'bottom-heavy'.

$\square$

B.5 General recommender function

To relax the contest success function that represents the recommender system, observing that an increase in σ is identical to a right shift in the distribution of consumption utility. This is akin to a first order stochastic dominance. Hence, allow a general recommender technology which depends on value-for-money. Then, without loss, consider a recommender technology $Q(\alpha_j, p_j)$ which first order stochastic dominates another recommender technology $L(\alpha_j, p_j)$ in consumption utility.

Proposition 10. *A monopolist platform prefers Q to L if and only if its per-transaction revenue using Q is sufficiently high such that* (7) *holds. This per-transaction revenue can be lower than that of L. When consumers are naive, the monopolist platform is less likely (compared to before) to prefer Q to L if and only if $E_Q[\alpha_j - p_j^*] > E_Q[\alpha_j - p_j^L]$.*

If the recommender system is sufficiently precise of consumption utility, then per-transaction revenues can be made up for by higher volume of sales. This is why the per-transaction revenue that leads the platform to prefer Q to L does not need to be as high as that of L. The results on naive consumers are in-line with those in the main text, showing that naive consumers lead the platform to become less likely to adopt the more precise recommender system.

Proof of Proposition 10. Consider the following recommender systems $Q(\alpha_j, p_j)$ and $L(\alpha_j, p_j)$ and without loss of generality, suppose Q first order stochastic dominates L such that $E_Q[\alpha_j - p_j] \geq E_L[\alpha_j - p_j]$. Let their PDFs be $q(\alpha_j, p_j)$ and $l(\alpha_j, p_j)$ respectively, which are both weakly increasing in consumption utility, $\alpha_j - p_j$. In other words, they are weakly increasing in the first argument, quality, and weakly decreasing in the second, price.

Recall consumers have a uniformly distributed cost of joining the platform. Then the profit function of any firm on the platform adopting the recommender system $\Gamma \in \{Q, L\}$ is $\int_0^1 \gamma(\alpha_h, p_h)u(\alpha_h, p_h)\, d\alpha_h\, \gamma(\alpha_j, p_j)p_j(1-r)$. Note that because the recommender functions can possibly assign a firm with quality α_j to receive a recommendation with probability 0, this is equivalent to saying that the integral is over all firm qualities, rather than the set of firms $\mathbf{N}$. Following the logic that an individual firm is unable to unilaterally influence consumer's decision to join the platform, this means a firm's profit maximizing price is $p_j^* = -\frac{\gamma(\alpha_j, p_j^*)}{\gamma_2'(\alpha_j, p_j^*)}$. Then the platform's profit function is $\Pi = -r \int_0^1 \gamma(\alpha_h, p_h^*)(\alpha_h + \frac{\gamma(\alpha_h, p_h^*)}{\gamma_2'(\alpha_h, p_h^*)})\, d\alpha_h \int_0^1 \frac{(\gamma(\alpha_j, p_j^*))^2}{\gamma_2'(\alpha_j, p_j^*)}\, d\alpha_j$. Note that $\int_0^1 \gamma(\alpha_h, p_h^*)(\alpha_h + \frac{\gamma(\alpha_h, p_h^*)}{\gamma_2'(\alpha_h, p_h^*)})\, d\alpha_h = E_\Gamma[\alpha_j - p_j^*]$.

Then the recommender system Q is preferred to L if

$$\begin{aligned} -rE_Q[\alpha_j - p_j^*] \int_0^1 \frac{(q(\alpha_h, p_h^*))^2}{q_2'(\alpha_h, p_h^*)}\, d\alpha_h &> -rE_L[\alpha_j - p_j^*] \int_0^1 \frac{(l(\alpha_h, p_h^*))^2}{l_2'(\alpha_h, p_h^*)}\, d\alpha_h \\ \Leftrightarrow -\int_0^1 \frac{(q(\alpha_h, p_h^*))^2}{q_2'(\alpha_h, p_h^*)}\, d\alpha_h &> -\int_0^1 \frac{(l(\alpha_h, p_h^*))^2}{l_2'(\alpha_h, p_h^*)}\, d\alpha_h \frac{E_L[\alpha_j - p_j^*]}{E_Q[\alpha_j - p_j^*]}, \end{aligned} \tag{7}$$

where $\frac{E_L[\alpha_j - p_j^*]}{E_Q[\alpha_j - p_j^*]} \in (0,1)$ and $-\int_0^1 \frac{(\gamma(\alpha_j, p_j^*))^2}{\gamma_2'(\alpha_j, p_j^*)}\, d\alpha_j$ is the average per-transaction revenue. Hence, if the average per-transaction revenue following Q is sufficiently large (and need not be larger than L), the platform prefers Q. Note it is possible for the per-transaction revenue of Q to be less than L and the platform would still prefer Q because more consumers join the platform if the recommender function sufficiently favors products which provide higher value-for-money.

Now, allow consumers to be naive such that they do not correctly anticipate the effect that the change in recommender technology has on firms pricing strategy.

Suppose, without loss of generality, that consumers believe firms pricing strategy always follows the recommender system L. Then the profit function of firms when the recommender system is L are $\int_0^1 l(\alpha_h, p_h) u(\alpha_h, p_h)\, d\alpha_h\, l(\alpha_j, p_j) p_j (1-r)$ and their corresponding pricing strategy are $p_j^L = -\frac{l(\alpha_j, p_j^L)}{l_2'(\alpha_j, p_j^L)}$. The platform's profit function is $\Pi^L = -r \int_0^1 l(\alpha_h, p_h^L)(\alpha_h + \frac{l(\alpha_h, p_h^L)}{l_2'(\alpha_h, p_h^L)})\, d\alpha_h \int_0^1 \frac{(l(\alpha_j, p_j^L))^2}{l_2'(\alpha_j, p_j^L)}\, d\alpha_j$.

When the recommender system is Q, because consumers are naive, $n^Q = \int_0^1 q(\alpha_h, p_h^L) u(\alpha_h, p_h^L)\, d\alpha_h$. Hence, the profit function of firms are $\int_0^1 q(\alpha_h, p_h^L) u(\alpha_h, p_h^L)\, d\alpha_h\, q(\alpha_j, p_j) p_j (1-r)$. However, because firms cannot unilaterally influence consumer's decision to join the platform, their profit maximizing price is $p_j^Q = -\frac{q(\alpha_j, p_j^Q)}{q_2'(\alpha_j, p_j^Q)}$. The platform's profit function is now $\Pi^Q = -r \int_0^1 q(\alpha_h, p_h^L)(\alpha_h + \frac{l(\alpha_h, p_h^L)}{l_2'(\alpha_h, p_h^L)})\, d\alpha_h \int_0^1 \frac{(q(\alpha_j, p_j^Q))^2}{q_2'(\alpha_j, p_j^Q)}\, d\alpha_j$.

Then the recommender system Q is preferred to L if

$$\begin{aligned} -rE_Q[\alpha_j - p_j^L] \int_0^1 \frac{(q(\alpha_h, p_h^Q))^2}{q_2'(\alpha_h, p_h^Q)}\, d\alpha_h &> -rE_L[\alpha_j - p_j^L] \int_0^1 \frac{(l(\alpha_h, p_h^L))^2}{l_2'(\alpha_h, p_h^L)}\, d\alpha_h \\ \Leftrightarrow -\int_0^1 \frac{(q(\alpha_h, p_h^Q))^2}{q_2'(\alpha_h, p_h^Q)}\, d\alpha_h &> -\int_0^1 \frac{(l(\alpha_h, p_h^L))^2}{l_2'(\alpha_h, p_h^L)}\, d\alpha_h \frac{E_L[\alpha_j - p_j^L]}{E_Q[\alpha_j - p_j^L]}. \end{aligned}$$

In other words, if $E_Q[\alpha_j - p_j^*] > E_Q[\alpha_j - p_j^L]$ then the platform is less likely to prefer recommender system Q to L when compared to non-naive consumers.

□

B.5.1 Specific functional forms

In this section, I consider two specific functional forms which are common in the literature. While these functional forms yield qualitatively similar trade-offs for a monopolist platform, their complexity can already be seen in the monopolist setting. Hence, for tractability, I do not consider these functional forms in a setting with competing platforms.

Tullock contest with utility exponent. In this section, I define $\lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma) \equiv \frac{(\alpha_j - p_j)^\sigma}{\int_{h \in \mathbf{N}} (\alpha_h - p_h)^\sigma)\, d\alpha_h}$, $\sigma > 0$. This formulation takes the familiar form of the Tullock contest success function with exponent parameter. A larger σ corresponds to recommender systems more precise of value-for-money, and $\sigma = 1$ represents a recommender system which is exactly based on value-for-money.

Consider firms pricing strategy, and recall that the mass of consumers joining the platform is $n = E[u]$ given by (1) such that individual firms are unable to influence the mass of consumers joining the platform. The firms profit function is $n(1-r)\frac{(\alpha-p)^\sigma}{\int_{h\in\mathbf{N}}(\alpha_h-p_h)^\sigma\, d\alpha_h}p$, and the profit maximizing pricing strategy is $p^* = \frac{\alpha}{1+\sigma}$. This means that all firms are always active on the platform, and because prices are decreasing in σ and higher σ means better match quality, the mass of consumers active on the platform increases in σ.

The mass of consumers joining the platform is therefore given by $n = \frac{\sigma}{2+\sigma}$ and consumer surplus is given by $\frac{\sigma^2}{2(2+\sigma)^2}$. Consumer surplus is increasing in σ.

The platform's profit function is therefore $\frac{\sigma}{2+\sigma}r\frac{1}{2+\sigma}$, which is maximized at $\sigma = 2$.

Therefore, like the main model, higher levels of σ causes firms to set lower prices and the platform prefers recommender systems which are more precise of value-for-money, which provides more consumer surplus than recommender systems that are purely based on value-for-money.

Logistic contest success function. In this section, I follow Casner and Teh (Forthcoming) and define $\lambda(\alpha_j, p_j, \mathbf{p}_{\text{-}j}, \sigma) \equiv \frac{\exp(\frac{\alpha_j - p_j}{\sigma})}{\int_{h\in\mathbf{N}}\exp(\frac{\alpha_h-p_h}{\sigma})\, d\alpha_h}$, allowing the platform to set any $\sigma \in \mathbf{R}_+$, where $\sigma = 1$ corresponds to matching based purely on value-for-money, and $\sigma \to \infty$ corresponds to completely random matching, and a lower level of σ reflects more precise recommender systems.

Consider firms pricing strategy, and recall that the mass of consumers joining the platform is $n = E[u]$, given by (1), such that firms are unable to influence the mass of consumers joining the platform. Therefore, firms optimal prices are $p^* = \sigma$. Since firms want to make positive profits, they choose to sell only if $p^* > 0$. Further, because consumers have free returns, only firms with quality $\alpha > \sigma$ sell on the platform and all other firms exit. Therefore, the platform's strategy is constrained to $\sigma \in (0, 1]$. Then the mass of consumers choosing to enter the market evaluates to $n = \frac{\exp(1/\sigma)(1-\sigma)}{\exp(1/\sigma)-\exp} - \sigma$. And the platform's profit function is $rn\int_\sigma^1 \frac{\exp(\alpha_h/\sigma)}{(\exp(1/\sigma)-\exp)\sigma}\sigma\, d\alpha_h = rn\sigma$. The platform's profit function is represented in Figure 3, and the optimal recommender system specification is $\sigma = 0.413$. This means, as in the main model, the platform prefers its recommender system to be more precise than value-for-money alone.

Qualitatively, the platform's trade-offs are similar to the main model. More precise recommender systems leads to lower per-transaction revenue, but can attract more consumers to the platform and increase the volume of transactions. However, there exists a screening effect that works in contrary to the main model. Whereas more precise recommendations lead to more screening in the main model, here, less precise recommender systems drive screening. This is because screening in the main model is driven by firms needing to set negative prices to obtain demand. And here, screening is driven by prices being too high when the recommender system is uninformative, such that consumers return all low quality products.

As in the main model, the presence of naive consumers means the platform is less able to use precise recommendations to influence consumers' decision to join the platform, and hence prefers to improve its profit by developing less precise recommendations to raise prices instead.

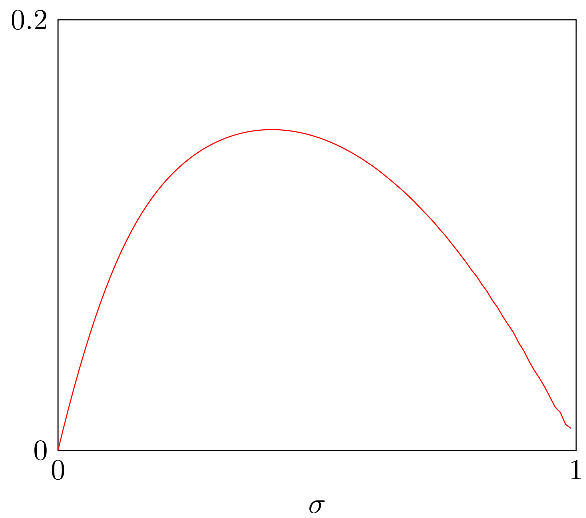

Figure 3: Platform profit function.

B.6 Asymmetric consumer cost

From the base model, allow for the following modification: Suppose consumer cost for each platform is drawn from different distributions. For tractability, let the cost of the incumbent platform $c_{i,I}$ be drawn from a uniform distribution R with support $[0, 1]$ and the cost of the entrant $c_{i,E}$ be drawn from a triangular T distribution with support $[0, 1]$ with peak 1. Proposition 11 summarizes the results of this modification.

Proposition 11. *There exists a unique equilibrium where* $\sigma_E = 0.547 > \sigma_I = 0.335$.

This shows that the platform facing higher consumer cost prefers to develop a more precise recommender system. Intuitively, this is because the entrant has to create more incentives for consumers to join the platform. Hence, this platform is more willing to give up more per-transaction revenue than the incumbent to attract consumers.

Proof of Proposition 11. The expected consumption utility from joining either platform is $\frac{1+2\sigma_k}{3}$ $\forall k \in \{I, C\}$. The mass of consumers joining either platform is given by

$$\begin{aligned} n_I &= \int_0^{E[u_E]} R(E[u_I] - E[u_E] + c_{i,E})t(c_{i,E})\, dc_{i,E} + R(E[u_I])(1 - T(E[u_E])) \\ &= \frac{26 + 54\sigma_I - 6\sigma_E - 12\sigma_E^2 - 8\sigma_E^3}{81} \\ n_E &= \int_0^{E[u_I]} T(E[u_E] - E[u_I] + c_{i,I})r(c_{i,I})\, dc_{i,I} + T(E[u_E])(1 - R(E[u_I])) \\ &= \frac{7 + 36\sigma_E^2 - 6\sigma_I + 8\sigma_I^3 + 6\sigma_E(5 - 4\sigma_I - 4\sigma_I^2)}{81} \end{aligned}$$

Since the firms' pricing strategy is independent of the mass of consumers joining the platform, $p^*_{j,k} = \frac{\alpha_j - \sigma_k}{2}$ $\forall k \in \{I, C\}$. And the per-transaction revenue for a platform k is $r\frac{1-\sigma_k}{3}$. Thus, on the incumbent

On the incumbent,

$$\Pi_I = r\frac{26 + 54\sigma_I - 6\sigma_E - 12\sigma_E^2 - 8\sigma_E^3}{81}\frac{1-\sigma_I}{3}$$

and we can solve that $\sigma_I = \frac{14+3\sigma_E+6\sigma_E^2+4\sigma_E^3}{54}$ maximizes profits.

On the entrant,

$$\Pi_E = r\frac{7 + 36\sigma_E^2 - 6\sigma_I + 8\sigma_I^3 + 6\sigma_E(5 - 4\sigma_I - 4\sigma_I^2)}{81}\frac{1-\sigma_E}{3}$$

and $\sigma_E = \frac{1+4\sigma_I+4\sigma_I^2 \pm \sqrt{70-46\sigma_I-48\sigma_I^2+8\sigma_I^3+16\sigma_I^4}}{18}$ maximizes its profit. Solving for σ_I and σ_E, we can show that the negative solution for σ_E does not satisfy the constraints of $\sigma_E \in [0, 1)$. And it is possible to obtain the numerical solutions $\sigma_I = 0.335$ and $\sigma_E = 0.547$.

□